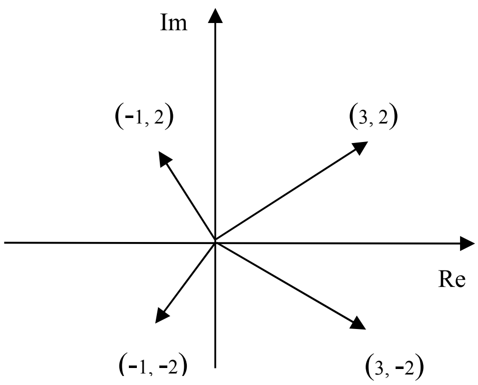
Question	Scheme	Marks	AOs
3	$z = 3 - 2\text{i}$ is also a root	B1	1.2
	$(z-(3+2\text{i}))(z-(3-2\text{i})) = \ldots$ **or** Sum of roots = 6, Product of roots = 13 $\Rightarrow \ldots$	M1	3.1a
	$= z^2 - 6z + 13$	A1	1.1b
	$(z^4 + az^3 + 6z^2 + bz + 65) = (z^2 - 6z + 13)(z^2 + cz + 5) \Rightarrow c = \ldots$	M1	3.1a
	$z^2 + 2z + 5 = 0$	A1	1.1b
	$z^2 + 2z + 5 = 0 \Rightarrow z = \ldots$	M1	1.1a
	$z = -1 \pm 2\text{i}$	A1	1.1b
		B1 $3 \pm 2\text{i}$ Plotted correctly	1.1b
		B1ft $-1 \pm 2\text{i}$ Plotted correctly	1.1b
			(9 marks)

Notes:

B1: Identifies the complex conjugate as another root
M1: Uses the conjugate pair and a correct method to find a quadratic factor
A1: Correct quadratic
M1: Uses the given quartic and their quadratic to identify the value of c
A1: Correct 3TQ
M1: Solves their second quadratic
A1: Correct second conjugate pair
B1: First conjugate pair plotted correctly and labelled
B1ft: Second conjugate pair plotted correctly and labelled (Follow through their second conjugate pair)

Paper 2: Core Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$\alpha+\beta+\gamma=8, \quad \alpha\beta+\beta\gamma+\gamma\alpha=28, \quad \alpha\beta\gamma=32$	B1	3.1a
	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$=\frac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha+2)(\beta+2)(\gamma+2)=(\alpha\beta+2\alpha+2\beta+4)(\gamma+2)$	M1	1.1b
	$=\alpha\beta\gamma+2(\alpha\beta+\alpha\gamma+\beta\gamma)+4(\alpha+\beta+\gamma)+8$	A1	1.1b
	$=32+2(28)+4(8)+8=128$	A1	1.1b
		(3)	
	Alternative:		
	$(x-2)^3-8(x-2)^2+28(x-2)-32=0$	M1	1.1b
	$=...-8+...-32+...-56-32=-128$	A1	1.1b
	$\therefore(\alpha+2)(\beta+2)(\gamma+2)=128$	A1	1.1b
		(3)	
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$	M1	3.1a
	$=8^2-2(28)=8$	A1ft	1.1b
		(2)	
			(8 marks)

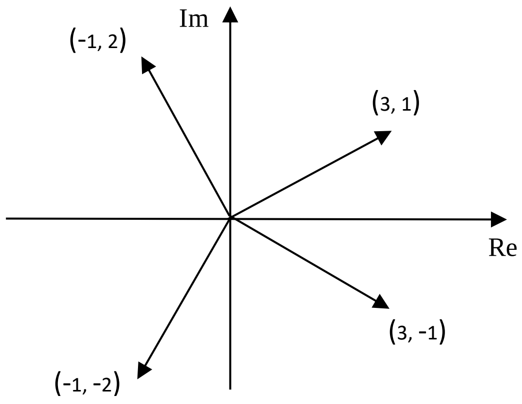
Notes:

(i)
B1: Identifies the correct values for all 3 expressions (can score anywhere)
M1: Uses a correct identity
A1ft: Correct value (follow through their 8, 28 and 32)

(ii)
M1: Attempts to expand
A1: Correct expansion
A1: Correct value

Alternative:
M1: Substitutes $x-2$ for x in the given cubic
A1: Calculates the correct constant term
A1: Changes sign and so obtains the correct value

(iii)
M1: Establishes the correct identity
A1ft: Correct value (follow through their 8, 28 and 32)

Question	Scheme		Marks	AOs
1(a)	$z = -1-2\text{i}$ or $z = 3+\text{i}$		M1	1.2
	$z = -1-2\text{i}$ and $z = 3+\text{i}$		A1	1.1b
			B1 B1	1.1b 1.1b
			(4)	
(b) **Way 1**	$(z-(-1+2\text{i}))(z-(-1-2\text{i}))=\ldots$ or $(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$	$\text{f}(z)=(z-(-1+2\text{i}))(z-(-1-2\text{i}))$ $(z-(3+\text{i}))(z-(3-\text{i}))=\ldots$	M1	3.1a
	z^2+2z+5 or $z^2-6z+10$	e.g. $\text{f}(z)=(z^2+2z+5)(\ldots)$	A1	1.1b
	z^2+2z+5 and $z^2-6z+10$	$\text{f}(z)=(z^3+z^2(-1-\text{i})+z(-1+2\text{i})-15-5\text{i})(\ldots)$	A1	1.1b
	$\text{f}(z)=(z^2+2z+5)(z^2-6z+10)$	Expands the brackets to forms a quartic	M1	3.1a
	$\text{f}(z)=z^4-4z^3+3z^2-10z+50$ or States $a=-4, b=3, c=-10, d=50$		A1	1.1b
			(5)	

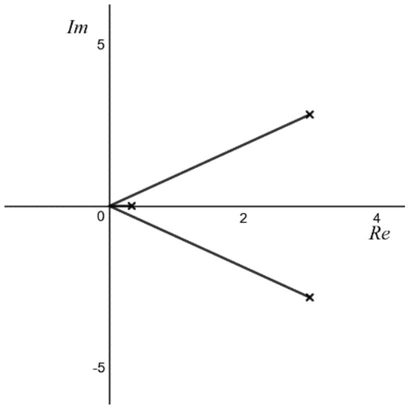
Question	Scheme	Marks	AOs
2(i)	$p+q+r=2, \quad pq+pr+qr=4, \quad pqr=5$	B1	3.1a
	$\frac{2}{p}+\frac{2}{q}+\frac{2}{r}=\frac{2(pq+pr+qr)}{pqr}$	M1	1.1b
	$=\frac{8}{5}$	A1ft	1.1b
		(3)	
	Alternative for part (i)		
	$x=\frac{2}{y}\Rightarrow\frac{8}{y^3}-\frac{8}{y^2}+\frac{8}{y}-5=0\Rightarrow 5y^3-8y^2+8y-8=0$	B1	3.1a
	$\frac{2}{p}+\frac{2}{q}+\frac{2}{r}=-\frac{-8}{5}$	M1	1.1b
	$=\frac{8}{5}$	A1ft	1.1b
		(3)	
(ii)	$(p-4)(q-4)(r-4)=(pq-4p-4q+16)(r-4)$ $=pqr-4pq-4pr-4qr+16p+16q+16r-64$	M1 A1	1.1b 1.1b
	$\left(=pqr-4(pq+pr+qr)+16(p+q+r)-64\right)$		
	$=5-4(4)+16(2)-64=-43$	A1	1.1b
		(3)	
	Alternative for part (ii)		
	$(x+4)^3-2(x+4)^2+4(x+4)-5=0$	M1	1.1b
	$=\ldots64+\ldots-32+\ldots16+\ldots-5=43$	A1	1.1b
	$\therefore(p-4)(q-4)(r-4)=-43$	A1	1.1b
		(3)	
(iii)	E.g. $p^3+q^3+r^3=$ $=(p+q+r)^3-3(p+q+r)(pq+pr+qr)+3pqr$ or $=(p+q+r)\left((p+q+r)^2-2(pq+pr+qr)-pq-pr-qr\right)+3pqr$ or $=2\left((p+q+r)^2-2(pq+pr+qr)\right)-4(p+q+r)+3pqr$ $\Rightarrow p^3+q^3+r^3=\ldots$	M1	3.1a
	$=2^3-3(2)(4)+3(5)=-1$ $=2\left(2^2-3(4)\right)+3(5)=-1$ $=2\left(2^2-2(4)\right)-4(2)+3(5)=-1$	A1	1.1b
		(2)	

	Alternative for part (iii)		
	$p^3-2p^2+4p-5=0,\ q^3-2q^2+4q-5=0,\ r^3-2r^2+4r-5=0$ $p^3+q^3+r^3-2\left(p^2+q^2+r^2\right)+4\left(p+q+r\right)-15=0$ $p^3+q^3+r^3=2\left(\left(p+q+r\right)^2-2\left(pq+pr+qr\right)\right)-4\left(p+q+r\right)+15$ $\Rightarrow p^3+q^3+r^3=...$	M1	3.1a
	$=2\left(2^2-2\left(4\right)\right)-4\left(2\right)+15=-1$	A1	1.1b
		(2)	

(8 marks)

Notes

(i)
B1: Identifies the correct values for all 3 expressions (can score anywhere). Allow notation such as $\sum p,\ \sum pq$ for the sum and pair sum.
M1: Uses a correct identity for the sum
A1ft: Correct value (follow through their 2, 4 and 5)
Alternative:
B1: Obtains the correct cubic in "*y*"
M1: Uses a correct method
A1ft: Correct value (follow through their 2, 4 and 5)
(ii)
M1: Attempt to expand – must have an expression that involves the sum, pair sum and product
A1: Correct expansion
A1: Correct value
Alternative:
M1: Substitutes $x + 4$ for x in the given cubic
A1: Calculates the correct constant term
A1: Correct value
(iii)
M1: Establishes a correct identity that is in terms of the sum, pair sum and product and substitutes to reach a numerical expression for $p^3+q^3+r^3$
A1: Correct value

Question	Scheme	Marks	AOs
1(a)	$\beta = 3+2\sqrt{2}\,\mathrm{i}$ is also a root	B1	1.2
	$\alpha\beta = 17, \alpha+\beta = 6$	B1	1.1b
	$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{57}{3}$	M1	1.1b
	$\alpha\gamma+\beta\gamma = \frac{57}{3} - 17 = \gamma(\alpha+\beta) = 6\gamma \Rightarrow \gamma = ...$	M1	3.1a
	$\gamma = \frac{1}{3}$	A1	2.2a
		B1 B1ft	1.1b 1.1b
		(7)	
	(a) Alternative:		
	$\beta = 3+2\sqrt{2}\,\mathrm{i}$ is also a root	B1	1.2
	$\left(z-\left(3+2\sqrt{2}\,\mathrm{i}\right)\right)\left(z-\left(3-2\sqrt{2}\,\mathrm{i}\right)\right) = z^2-6z+17$	B1	1.1b
	$\mathrm{f}(z) = (z^2-6z+17)(3z+a) = 3z^3+az^2-18z^2-6az+51z+17a$	M1	1.1b
	$\Rightarrow 51-6a = 57 \Rightarrow a = -1 \Rightarrow \gamma = ...$	M1	3.1a
	$\gamma = \frac{1}{3}$	A1	2.2a
	Then B1 B1ft as above		
		(7)	
(b)	$3-2\sqrt{2}\,\mathrm{i}+3+2\sqrt{2}\,\mathrm{i}+\frac{1}{3} = -\frac{p}{3} \Rightarrow p = ...$ or $\left(3-2\sqrt{2}\,\mathrm{i}\right)\left(3+2\sqrt{2}\,\mathrm{i}\right)\times\frac{1}{3} = -\frac{q}{3} \Rightarrow q = ...$	M1	3.1a
	$p = -19$ **or** $q = -17$	A1	1.1b
	$p = -19$ **and** $q = -17$	A1	1.1b
		(3)	
	(b) Alternative:		
	$\mathrm{f}(z) = (z^2-6z+17)(3z-1) = 3z^3+pz^2+57z+q$	M1	3.1a

	$\Rightarrow p = ..., q = ...$		
	$p = -19$ **or** $q = -17$	A1	1.1b
	$p = -19$ **and** $q = -17$	A1	1.1b
		(3)	
			(10 marks)

Notes

(a)
B1: Identifies the correct complex conjugate as another root
B1: Correct values for the sum and product for the conjugate pair
M1: Correct application of the pair sum
M1: Identifies a complete and correct strategy for identifying the third root
A1: Deduces the correct third root
B1: $3 \pm 2\sqrt{2}\,\mathrm{i}$ plotted correctly, in quadrants 1 and 4 which are reflections in the real axis. Do not be concerned about labelling or scaling.
B1ft: Their real root plotted correctly, in correct relative position to the two complex roots. Scales are not needed but if correct, the real root must be close to the origin compared to the complex roots.

Alternative:
B1: Identifies the correct complex conjugate as another root
B1: Correct quadratic factor obtained
M1: Expands their quadratic×(3*z* + "*a*") or attempts to factor out the quadratic, or use long division, leading to a factor (3*z* + "*a*"). Implied by seeing $(z^2 - 6z + 17)(3z + a)$ with any value of *a* (or with their quadratic).
M1: Proceeds to extract the root from their third factor of from (3*z* + "*a*").
A1: Deduces the correct third root. If not explicitly stated, look for it on their diagram.
B1: $3 \pm 2\sqrt{2}\,\mathrm{i}$ plotted correctly, as above
B1ft: Their real root plotted correctly as above.

(b)
M1: Correct strategy used for identifying at least one of *p* or *q*
A1: At least one value correct
A1: Both values correct

Alternative:
M1: Correct strategy by expanding their quadratic and linear factors to identifying at least one of *p* or *q*
A1: At least one value correct
A1: Both values correct

Note: some may attempt to use the factor theorem with the complex root.
$\mathrm{f}\left(3 - 2\mathrm{i}\sqrt{2}\right) = 36 + p + q + \mathrm{i}\left(-228\sqrt{2} - 12\sqrt{2}p\right) = 0$
2nd B1: equate real and imaginary components to 0 to get correct equations
$36 + p + q = 0, -228\sqrt{2} - 12\sqrt{2}p = 0$
1st M1: solves their equations $\Rightarrow p = -19, q = -17$
2nd M1: Solves the cubic (may be from calculator). The 1st B1 may then be implied for the second complex root, and the rest as main scheme.

Question	Scheme	Marks	AOs
3	$w = 4x - 1 \Rightarrow x = \dfrac{w+1}{4}$	B1	3.1a
	$a\left(\dfrac{w+1}{4}\right)^3 + b\left(\dfrac{w+1}{4}\right)^2 - 19\left(\dfrac{w+1}{4}\right) - b\ (=0)$ or $(4x-1)^3 - 9(4x-1)^2 - 97(4x-1) + c\ (=0)$	M1	3.1a
	$aw^3 + (3a+4b)w^2 + (3a+8b-304)w + (a-60b-304) = 0$ or $64x^3 - 192x^2 - 304x + 87 + c = 0$	M1	1.1b
	Divides by *a* and equates the coefficients of w^2 and *w* $\dfrac{3a+4b}{a} = -9 \quad \dfrac{3a+8b-304}{a} = -97$ and solves simultaneously to find a value for *a* or a value for *b* **<u>Note:</u>** $12a + 4b = 0$ and $100a + 8b = 304$ or Divides through by '16' leading to values of *a* and *b* $4x^3 - 12x^2 - 19x + \dfrac{87+c}{19} = 0$	M1	3.1a
	$c = \dfrac{a-60b-304}{a} = \ldots$ or $\dfrac{87+c}{19} = 12$ Þ $c = \ldots$	M1	1.1b
	$a = 4 \quad b = -12 \quad c = 105$	A1	1.1b
		(6)	
			(6 marks)

Notes:

B1: Selects the method of making a connection between *x* and *w* by writing $w = 4x - 1$ or $x = \dfrac{w+1}{4}$

M1: Applies the process of substituting their $x = \dfrac{w+1}{4}$ into $ax^3 + bx^2 - 19x - b = 0$ or $w = 4x - 1$ into $w^3 - 9w^2 - 97w + c = 0$. Must be substitution of the correct variable into the opposing equation but may be scored if the initial linear equation is incorrect (e.g. $x = 4w - 1$ into the first equation). Note that the " = 0 " can be missing for this mark.

M1: Expands the brackets and collects terms in their equation (in *x* or *w*). Note that the " = 0 " can be missing for this mark.

M1: A complete method for finding a value for *a* or *b*. See scheme, it involves dividing through by an appropriate factor for their equation to balance the w^3 or $-19x$ terms, then equating other coefficients and solving equations if necessary.

M1: A complete method for finding a value for *c*. They must have divided through by an appropriate factor as per the previous M before attempting to compare the constant coefficient (and use their *a* and *b* if appropriate).

A1: $a = 4 \quad b = -12 \quad c = 105$

Question	Scheme	Marks	AOs
6(a)	$4x^3 + px^2 - 14x + q = 0 \Rightarrow x^3 + \frac{p}{4}x^2 - \frac{14}{4}x + \frac{q}{4} = 0$ $\alpha + \beta + \gamma = -\frac{p}{4}$ $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{14}{4}$ or $-\frac{7}{2}$	B1	3.1a
	$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ $\left(-\frac{p}{4}\right)^2 = 16 + 2\left(-\frac{7}{2}\right) \Rightarrow p = \ldots$ or $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) = \alpha^2 + \beta^2 + \gamma^2$ $\left(-\frac{p}{4}\right)^2 - 2\left(-\frac{7}{2}\right) = 16 \Rightarrow p = \ldots$	M1	3.1a
	$p = 12$ * cso	A1*	1.1b
		(3)	
(b)	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$\frac{\left(-\frac{7}{2}\right)}{\left(\frac{-q}{4}\right)} = \frac{14}{3} \Rightarrow q = \ldots$	M1	1.1b
	$q = 3$	A1	1.1b
		(3)	
	Alternative $4\left(\frac{1}{w}\right)^3 + 12\left(\frac{1}{w}\right)^2 - 14\left(\frac{1}{w}\right) + q\{= 0\}$	M1	1.1b
	$qw^3 - 14w^2 + 12w + 4 = 0 \Rightarrow \frac{14}{3} = -\frac{-14}{q} \Rightarrow q = \ldots$	M1	1.1b
	$q = 3$	A1	1.1b
		(3)	
(c)	$(\alpha - 1)(\beta - 1)(\gamma - 1) = \ldots$ $= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$	M1 A1	1.1a 1.1b
	$= \left(-\frac{\text{their } 3}{4}\right) - \left(-\frac{7}{2}\right) + \left(-\frac{12}{4}\right) - 1 = \ldots$	dM1	1.1b
	$= -\frac{5}{4}$	A1	1.1b
		(4)	
Alt	$4(x + 1)^3 + 12(x + 1)^2 - 14(x + 1) + \text{'3'}\{= 0\}$ or substitutes in 1	M1	1.1a
	$= \ldots 4 + \ldots 12 + \ldots -14 + \text{'3'} = 5$ or $4x^3 + 24x^2 + 22x + 2 +$ 'their q'	A1ft	1.1b

	$= -\frac{\text{'their constant'}}{4}$	dM1	1.1b
	$= -\frac{5}{4}$	A1	1.1b
(10 marks)			
Notes:			

(a)

B1: Identifies the correct values for the sum and pair sum. This may be implied by substituting into an equation, it must be clear

M1: Uses the correct identity and values of their sum **and** their pair sum to find a value of p

A1*: $p = 12$ cso there is no need to see a reason

(b)

M1: Establishes a correct identity

M1: Uses their identity and their pair sum and their product of roots to find a value of q. Condone a slip but the intention must be clear.

A1: $q = 3$ Allow this mark from incorrect sign of both pair sum and product

Alternative

M1: Uses $x = \frac{1}{w}$ the substitution

M1: Simplifies to an quartic equation of the form $aw^3 + bw^2 + cw + d = 0$ and uses $\frac{14}{3} = -\frac{b}{a}$ to find a value for q

A1: $q = 3$

(c)

M1: Attempts to multiply out the three brackets.

A1: Correct expansion.

dM1: Dependent on previous method**.** Substitutes in the value of their sum, pair sum and the value of their product as appropriate. Condone a slip but the intention must be clear

A1: Correct value

Alternative

M1: Substitutes $(x + 1)$ or $x = 1$ into the cubic with their value of q. Allow the use of different letters e.g. $(w + 1)$

A1ft: Correct constant terms, follow through on their value of q

dM1: Applies $-\frac{\text{'their constant'}}{4}$

A1: Correct value

Question	Scheme	Marks	AOs
1	$\{w = x+2 \Rightarrow\}\ x = w-2$	**B1**	3.1a
	$(w-2)^3 - 7(w-2)^2 - 12(w-2) + 6\ (=0)$	**M1**	1.1b
	$(w^3 - 6w^2 + 12w - 8) - 7(w^2 - 4w + 4) - 12(w-2) + 6$ $w^3 - 6w^2 + 12w - 8 - 7w^2 + 28w - 28 - 12w + 24 + 6$ $= w^3 + \ldots w^2 + \ldots w + \ldots$	**M1**	3.1a
	$w^3 - 13w^2 + 28w - 6 = 0$	**A1** **A1**	1.1b 1.1b
		(5)	
	Alternative using sum, pair sum and product of roots:		
	$\alpha + \beta + \gamma = 7,\ \alpha\beta + \beta\gamma + \alpha\gamma = -12,\ \alpha\beta\gamma = -6$	**B1**	3.1a
	New sum: $\alpha + 2 + \beta + 2 + \gamma + 2 = (\alpha + \beta + \gamma) + 6 = 7 + 6 = 13$ New pair sum: $(\alpha+2)(\beta+2) + (\alpha+2)(\gamma+2) + (\beta+2)(\gamma+2)$ $= (\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 12 = -12 + 4\times 7 + 12 = 28$ New product: $(\alpha+2)(\beta+2)(\gamma+2)$ $= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 8$ $= -6 + 2\times -12 + 4\times 7 + 8 = 6$	**M1**	3.1a
	$p = -\text{"13"},\ q = 28,\ r = -\text{"6"}$ **or** $w^3 - \text{"13"}w^2 + \text{"28"}w - \text{"6"}\ (=0)$	**M1**	1.1b
	$w^3 - 13w^2 + 28w - 6 = 0$	**A1** **A1**	1.1b 1.1b
		(5 marks)	

Notes:

Allow a variable other than *w* to be used for the first 4 marks.

<u>The "= 0" is not required until the final mark.</u>

B1: Selects the method of making a connection between *x* and *w* by writing $x = w-2$

M1: Applies the process of substituting their $x = \text{"}w-2\text{"}$ into the equation for all occurrences of *x*.

M1: Depends on having attempted substituting either $x = w-2$ or $x = w+2$ into the equation. This mark is for manipulating their resulting equation into the required form so must have gathered terms. Condone poor squaring/cubing of brackets as long as a cubic expression is obtained.

A1: At least two of *p*, *q* and *r* correct.

A1: Correct final <u>equation</u> (including "= 0"). **Must be an equation in *w*.**

Note if they say e.g. *x* = *w* – 2 and then substitute *w* + 2, it is possible to score B1 M0 M1

Note if they say e.g. *x* = *w* + 2 and then substitute *w* – 2, allow recovery

Alternative:

B1: Selects the method of giving three correct equations for the sum, pair sum and product in terms of α, β and γ. Note that the correct values may be seen embedded when they attempt the new sum, pair sum and product e.g. $(\alpha+2)(\beta+2)(\gamma+2) = \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha+\beta+\gamma) + 8$

$$= \underline{-\mathbf{6}} + 2(\underline{\mathbf{-12}}) + 4(\underline{\mathbf{7}}) + 8$$

M1: Applies the process of finding the new sum, pair sum and product. Mark positively here and allow slips provided they are attempting $\alpha + 2 + \beta + 2 + \gamma + 2,\ (\alpha+2)(\beta+2) + (\alpha+2)(\gamma+2) + (\beta+2)(\gamma+2)$ and $(\alpha+2)(\beta+2)(\gamma+2)$

M1: In this method, this mark is for choosing *p* = – (their new sum), *q* = their new pair sum, *r* = – (their new product) **or** forming $w^2 - (\text{new sum})w^2 + (\text{new pair sum})w - (\text{new product})$

A1: At least two of *p*, *q* and *r* correct. As values or seen in their equation.

A1: Correct final <u>equation</u> (including "= 0"). **Must be an equation in *w*.**

In all methods, the final A mark depends on all the previous marks.

Question	Scheme		Marks	AOs
8(a)	The real axis. Horizontal line through (0, 0) Line $y = 0$ Accept on a diagram	The other possibility is that all three roots have the same real part so lie on a vertical line/perpendicular to the real axis/parallel to the imaginary axis Line $x = k$ where k is a real number Accept on a diagram	**B1** **B1**	3.1a 2.2a
			(2)	
(b)	Other roots are $\frac{3}{2}$ and $\frac{3}{2}-\frac{3}{2}\text{i}$		**B1**	3.2a
			(1)	
(c)(i)	Common root must be $\frac{3}{2}$		**B1**	2.2a
			(1)	
(ii)	Sets product of roots = – 12 using their $\frac{3}{2}\times -4\times\alpha = -12$ Or $\text{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z-\alpha)$		**M1**	1.1b
	Solves to find a value of the third root their $\frac{3}{2}\times -4\times\alpha = -12 \Rightarrow \alpha = 2$ Or $\text{g}(z)=\left(z-\frac{3}{2}\right)(z\pm 4)(z-\alpha)\Rightarrow -\frac{3}{2}\times 4\times -\alpha = 12 \Rightarrow \alpha = 2$		**M1** **A1**	3.1a 1.1b
			(3)	
(d)	$8\left\{z-\frac{3}{2}\right\}\left(z-\frac{3}{2}-\frac{3}{2}\text{i}\right)\left(z-\frac{3}{2}+\frac{3}{2}\text{i}\right)=8\left\{z-\frac{3}{2}\right\}\left(z^2-3z+\frac{9}{2}\right)$ Or $b=-8\left(\frac{3}{2}+\frac{3}{2}+\frac{3}{2}\text{i}+\frac{3}{2}-\frac{3}{2}\text{i}\right)=\ldots\{-36\}$ $c=8\left[\left(\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\right)+\left(\frac{3}{2}\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right)+\left(\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right)\right]=\ldots\{72\}$ $d=-8\left[\frac{3}{2}\times\left(\frac{3}{2}+\frac{3}{2}\text{i}\right)\times\left(\frac{3}{2}-\frac{3}{2}\text{i}\right)\right]=\ldots\{-54\}$		**M1**	1.1b

	$f(z)=g(z)\Rightarrow 8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)=\left(z-\frac{3}{2}\right)(z+4)(z-2)$ $\Rightarrow 8z^2-24z+36=(z+4)(z-2)$ Or Either $g(z)=\left(z-\frac{3}{2}\right)(z+4)(z-2)=\ldots$ or $P=-\left(\frac{3}{2}-4+2\right)=\ldots\left\{\frac{1}{2}\right\}$ and $Q=\left(\frac{3}{2}\times -4\right)+\left(\frac{3}{2}\times 2\right)+(2\times -4)=\ldots\{-11\}$ to find g(*z*) and sets their f(*z*) = their g(*z*) $8z^3-36z^2+72z-54=z^3+\frac{1}{2}z^2-11z+12$	**M1**	3.1a
	$7z^2-26z+44=0\Rightarrow z=\ldots$ or $7z^3-\frac{73}{2}z^2+83z-66=0\Rightarrow z=\ldots$	**M1**	1.1b
	So solutions are $\frac{3}{2}$, $\frac{13\pm i\sqrt{139}}{7}$	**A1**	1.1b
		(4)	

(11 marks)

Notes:

(a)

B1: One correct line described

B1: Two correct lines described

Special case: If candidate states that "any line is possible" score B1B1 (as they may be considering a cubic with complex coefficients).

(b)

B1: Interprets the conclusion from (a) in context by identifying the correct two roots.

(c)

B1: Deduces the real root is the one in common.

M1: Sets product of roots = – 12 using their $\frac{3}{2}\times -4\times\alpha=-12$. Alternatively forms an equation for g(*z*) using the roots

M1: Solves their equation to find the third root, condone use of 12 for this mark. Alternatively multiply their constant and sets = 12 to find the third root. Condone a sign slip for this mark

A1: Correct third root

(d)

M1: Uses their roots of f(*z*) to form a cubic expression for f(*z*), and expands to at least a linear term times a

Question	Scheme	Marks	AOs
2	$w=2z+1\Rightarrow z=\frac{w-1}{2}$	B1	3.1a
	$\left(\frac{w-1}{2}\right)^3-3\left(\frac{w-1}{2}\right)^2+\left(\frac{w-1}{2}\right)+5=0$	M1	3.1a
	$\frac{1}{8}\left(w^3-3w^2+3w-1\right)-\frac{3}{4}\left(w^2-2w+1\right)+\frac{w-1}{2}+5=0$		
	$w^3-9w^2+19w+29=0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
ALT 1	$\alpha+\beta+\gamma=3, \alpha\beta+\beta\gamma+\alpha\gamma=1, \alpha\beta\gamma=-5$	B1	3.1a
	New sum = $2\left(\alpha+\beta+\gamma\right)+3=9$	M1	3.1a
	New pair sum = $4\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)+4\left(\alpha+\beta+\gamma\right)+3=19$		
	New product = $8\alpha\beta\gamma+4\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)+2\left(\alpha+\beta+\gamma\right)+1=-29$		
	$w^3-9w^2+19w+29=0$	M1 A1 A1	1.1b 1.1b 1.1b
		(5)	
		(5 marks)	

Notes

B1: Selects the method of making a connection between z and w by writing $z=\frac{w-1}{2}$

M1: Applies the process of substituting their $z=\frac{w-1}{2}$ into $z^3-3z^2+z+5=0$

(Allow $z = 2w + 1$)

M1: Manipulates their equation into the form $w^3+pw^2+qw+r\left(=0\right)$ having substituted their z in terms of w. Note that the "= 0" can be missing for this mark.

A1: At least two of p, q, r correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0"

The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w.

ALT1

B1: Selects the method of giving three correct equations containing α, β and γ

M1: Applies the process of finding the new sum, new pair sum, new product

M1: Applies $w^3-\left(\text{new sum}\right)w^2+\left(\text{new pair sum}\right)w-\left(\text{new product}\right)\left(=0\right)$

or identifies p as –(new sum) q as (new pair sum) and r as –(new product)

A1: At least two of p, q, r correct.

A1: Fully correct equation including "= 0"

The first 4 marks are available if another letter is used instead of w but the final answer must be in terms of w.

Question	Scheme	Marks	AOs
2.	$\{w = x+3 \Rightarrow\}\ x = w-3$	B1	3.1a
	$2(w-3)^3 + 6(w-3)^2 - 3(w-3) + 12\ (=0)$	M1	1.1b
	$2w^3 - 18w^2 + 54w - 54 + 6(w^2 - 6w + 9) - 3w + 9 + 12 (=0)$		
	$2w^3 - 12w^2 + 15w + 21 = 0$ (So $p = 2$, $q = -12$, $r = 15$ and $s = 21$)	M1 A1 A1	3.1a 1.1b 1.1b
		(5)	
ALT 1	$\alpha + \beta + \gamma = -\frac{6}{2} = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -\frac{3}{2},\ \alpha\beta\gamma = -\frac{12}{2} = -6$	B1	3.1a
	sum roots $= \alpha + 3 + \beta + 3 + \gamma + 3$ $= \alpha + \beta + \gamma + 9 = -3 + 9 = 6$ pair sum $= (\alpha+3)(\beta+3) + (\alpha+3)(\gamma+3) + (\beta+3)(\gamma+3)$ $= \alpha\beta + \alpha\gamma + \beta\gamma + 6(\alpha+\beta+\gamma) + 27$ $= -\frac{3}{2} + 6\times -3 + 27 = \frac{15}{2}$ product $= (\alpha+3)(\beta+3)(\gamma+3)$ $= \alpha\beta\gamma + 3(\alpha\beta + \alpha\gamma + \beta\gamma) + 9(\alpha+\beta+\gamma) + 27$ $= -6 + 3\times -\frac{3}{2} + 9\times -3 + 27 = -\frac{21}{2}$	M1	3.1a
	$w^3 - 6w^2 + \frac{15}{2}w - \left(-\frac{21}{2}\right)\ (=0)$	M1	1.1b
	$2w^3 - 12w^2 + 15w + 21 = 0$ (So $p = 2$, $q = -12$, $r = 15$ and $s = 21$)	A1 A1	1.1b 1.1b
		(5)	
	(5 marks)		

Notes

	B1	Selects the method of making a connection between x and w by writing $x = w - 3$
	M1	Applies the process of substituting their $x = aw \pm b$ into $2x^3 + 6x^2 - 3x + 12\ (=0)$ So accept e.g. if $x = \frac{w}{3}$ is used.
	M1	Depends on having attempted substituting either $x = w-3$ or $x = w+3$ into the equation. This mark is for manipulating their resulting equation into the form $pw^3 + qw^2 + rw + s(=0)$ ($p \neq 0$). The "= 0" may be implied for this.
See note	**A1**	At least three of p, q, r and s are correct in an equation with integer coefficients. (need not have "= 0")
	A1	Correct final equation, including "=0". Accept integer multiples.
ALT 1	**B1**	Selects the method of giving three correct equations each containing α, β and γ.
	M1	Applies the process of finding sum roots, pair sum and product.
	M1	Applies $w^3 -$ (their sum roots) $w^2 +$ (their pair sum)w $-$ (their product) (= 0) Must be correct identities, but if quoted allow slips in substitution, but the "=0" may be implied.
See note	**A1**	At least three of p, q, r and s are correct in an equation with integer coefficients. (need not have "=0")
	A1	Correct final equation, including "=0". Accept multiples with integer coefficients.

Note: may use another variable than w for the first four marks, but the final equation must be in terms of w

Notes: Do not isw the final two A marks – if subsequent division by 2 occurs then mark the final answer.

Question	Scheme	Marks	AOs
7. (a)	$\alpha+\beta+\left(\alpha+\dfrac{12}{\alpha}-\beta\right)=8$ so $2\alpha+\dfrac{12}{\alpha}=8$	M1 A1	1.1b 1.1b
	$\Rightarrow 2\alpha^2-8\alpha+12=0$ or $\alpha^2-4\alpha+6=0$ $\Rightarrow \alpha=\dfrac{4\pm\sqrt{(-4)^2-4(1)(6)}}{2(1)}$ or $(\alpha-2)^2-4+6=0 \Rightarrow \alpha=...$	M1	1.1b
	$\Rightarrow \alpha=2\pm i\sqrt{2}$ are the two complex roots	A1	1.1b
	A correct full method to find the third root. Common methods are: Sum of roots = 8 $\Rightarrow$ third root $=8-\left(2+i\sqrt{2}\right)-\left(2-i\sqrt{2}\right)=...$ third root $=2+i\sqrt{2}+\dfrac{12}{2+i\sqrt{2}}-\left(2-i\sqrt{2}\right)=...$ Product of roots = 24 $\Rightarrow$ third root $=\dfrac{24}{\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)}=...$ $(z-\alpha)(z-\beta)=z^2-4z+6\Rightarrow f(z)=\left(z^2-4z+6\right)(z-\gamma)\Rightarrow\gamma=...$ (or long division to find third factor).	M1	3.1a
	Hence the roots of $f(z)=0$ are $2\pm i\sqrt{2}$ and 4	A1	1.1b
		(6)	
(b)	E.g. f(4) = 0 $\Rightarrow 4^3-8\times4^2+4p-24=0\Rightarrow p=...$ Or $p=\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)+4\left(2+i\sqrt{2}\right)+4\left(2-i\sqrt{2}\right)\Rightarrow p=...$ Or $f(z)=(z-4)(z^2-4z+6)\Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=22$ **cso**	A1	1.1b
		(2)	
	(8 marks)		

Notes

(a)	**M1**	Equates sum of roots to 8 and obtains an equation in just α.
	A1	Obtains a correct equation in α.
	M1	Forms a three term quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$
	A1	$\alpha=2\pm i\sqrt{2}$
	M1	Any correct method for finding the remaining root. There are various routes possible. See scheme for common ones. Allow this mark if –24 is used as the product. See note below for a less common approach.
	A1	Third root found with all three roots correct. Note α and β need not be identified.
(b)	**M1**	Any correct method of finding p. For example, applies the factor theorem, process of finding the pair sum of roots, or uses the roots to form f(z).
	A1	$p=22$ by correct solution only. Note: this can be found using only their complex roots from (a) (e.g. by factor theorem)

Note for (a) final M – it is possible to find the second and third roots using only one initial root (e.g. if second root forgotten or error leads to only one initial root being found).

Product of roots = $\alpha\beta\left(\alpha+\dfrac{12}{\alpha}-\beta\right)=24\Rightarrow\alpha\beta^2-\left(\alpha^2+12\right)\beta+24=0$, substitutes in α and attempts to solve the quadratic in β to achieve remaining roots. The final M can be gained once three roots in total have been obtained. (This is unlikely to be seen as part of a correct answer.) Allow if −24 has been used for the product.

Question	Scheme	Marks	AOs
7	$z_2 = 2-3\text{i}$	B1	1.1b
	$(z_3 =)\, p-3\text{i}$ and $(z_4 =)\, p+3\text{i}$ May be seen in an Argand diagram	M1	3.1a
	$(z_3 =) -4-3\text{i}$ and $(z_4 =) -4+3\text{i}$ May be seen in an Argand diagram, but the complex numbers used in their method takes precedence	A1	1.1b
	$(z^2-4z+13)(z^2+8z+25)$ or $(z-(2-3\text{i}))(z-(2+3\text{i}))(z-(-4-3\text{i}))(z-(-4+3\text{i}))$ or $a = -[(2-3\text{i})+(2+3\text{i})+(-4-3\text{i})+(-4+3\text{i})]$ and $b=(2-3\text{i})(2+3\text{i})+(2-3\text{i})(-4-3\text{i})+(2-3\text{i})(-4+3\text{i}) +(2+3\text{i})(-4-3\text{i})+(2+3\text{i})(-4+3\text{i})+(-4-3\text{i})(-4+3\text{i})$ and $c=-\left[\begin{array}{l}(2-3\text{i})(2+3\text{i})(-4-3\text{i})+(2-3\text{i})(2+3\text{i})(-4+3\text{i})\\ +(2-3\text{i})(-4-3\text{i})(-4+3\text{i})+(2+3\text{i})(-4-3\text{i})(-4+3\text{i})\end{array}\right]$ and $d=(2-3\text{i})(2+3\text{i})(-4-3\text{i})(-4+3\text{i})$ or Substitutes in one root from each conjugate pair and equates real and imaginary parts and solves simultaneously $(2\pm3\text{i})^4+a(2\pm3\text{i})^3+b(2\pm3\text{i})^2+c(2\pm3\text{i})+d=0$ $(-4\pm3\text{i})^4+a(-4\pm3\text{i})^3+b(-4\pm3\text{i})^2+c(-4\pm3\text{i})+d=0$	dM1	3.1a
	$a=4, b=6, c=4, d=325$ $\text{f}(z)=z^4+4z^3+6z^2+4z+325$	A1 A1	1.1b 1.1b
		(6)	
			(6 marks)

Notes:

B1: Seen $2-3\text{i}$

M1: Finds the third and fourth roots of the form $p\pm3\text{i}$. May be seen in an Argand diagram

A1: Third and fourth roots are $-4\pm3\text{i}$. May be seen in an Argand diagram

dM1: Uses an appropriate method to find $\text{f}(z)$. If using roots of a polynomial at least 3 coefficients must be attempted.

A1: At least two of *a*, *b*, *c*, *d* correct

A1: All *a*, *b*, *c* and *d* correct

Question	Scheme	Marks	AOs
9(a)	$\alpha\beta\gamma = -\frac{1}{3}$ and $\alpha\beta+\alpha\gamma+\beta\gamma = -\frac{4}{3}$	B1	3.1a
	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = \frac{-4/3}{-1/3}$	M1	1.1b
	$= 4$	A1	1.1b
		(3)	
(b)	$\left\{\alpha+\beta+\gamma = -\frac{1}{3}\right\}$ New product $= \frac{1}{\alpha}\times\frac{1}{\beta}\times\frac{1}{\gamma} = \frac{1}{\alpha\beta\gamma} = \frac{1}{-1/3} = ...(-3)$ New pair sum $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma} = \frac{\gamma+\alpha+\beta}{\alpha\beta\gamma} = \frac{-1/3}{-1/3} = ...(1)$	M1	3.1a
	$x^3 - (\text{part (a)})x^2 + (\text{new pair sum})x - (\text{new product}) (= 0)$	M1	1.1b
	$x^3 - 4x^2 + x + 3 = 0$	A1	1.1b
		(3)	
	Alternative e.g. $z = \frac{1}{x} \Rightarrow \frac{3}{x^3}+\frac{1}{x^2}-\frac{4}{x}+1 = 0$	M1	3.1a
	$x^3 - 4x^2 + x + 3 = 0$	M1 A1	1.1b 1.1b
		(3)	
			(6 marks)

Notes:

(a)

B1: Correct values for the product and pair sum of the roots

M1: A complete method to find the sum of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. Must substitute in their values of the product and pair sum

A1: correct value 4

Note: If candidate does not divide by 3 so that $\alpha\beta\gamma = -1$ and $\alpha\beta+\alpha\gamma+\beta\gamma = -4$ the maximum they can score is B0 M1 A0

(b)

M1: A correct method to find the value of the new pair sum and the value of the new product

M1: Applies $x^3 - (\text{part (a)})x^2 + (\text{their new pair sum})x - (\text{their new product}) (= 0)$

A1: Fully correct equation, in any variable, including = 0

Question	Scheme	Marks	AOs
2	$w=3x-2 \Rightarrow x=\dfrac{w+2}{3}$	B1	3.1a
	$9\left(\dfrac{w+2}{3}\right)^3-5\left(\dfrac{w+2}{3}\right)^2+4\left(\dfrac{w+2}{3}\right)+7=0$	M1	3.1a
	$\dfrac{1}{3}\left(w^3+6w^2+12w+8\right)-\dfrac{5}{9}\left(w^2+4w+4\right)+\dfrac{4}{3}\left(w+2\right)+7=0$		
	$3w^3+13w^2+28w+91=0$	dM1 A1 A1	1.1b 1.1b 1.1b
		(5)	
	Alternative:		
	$\alpha+\beta+\gamma=\dfrac{5}{9},\alpha\beta+\beta\gamma+\alpha\gamma=\dfrac{4}{9},\alpha\beta\gamma=-\dfrac{7}{9}$	B1	3.1a
	New sum = $3\left(\alpha+\beta+\gamma\right)-6=-\dfrac{13}{3}$	M1	3.1a
	New pair sum = $9\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)-12\left(\alpha+\beta+\gamma\right)+12=\dfrac{28}{3}$		
	New product = $27\alpha\beta\gamma-18\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)+12\left(\alpha+\beta+\gamma\right)-8=-\dfrac{91}{3}$		
	$w^3-\left(-\dfrac{13}{3}\right)w^2+\dfrac{28}{3}w-\left(-\dfrac{91}{3}\right)=0$	dM1	1.1b
	$3w^3+13w^2+28w+91=0$	A1 A1	1.1b 1.1b
		(5)	
		(5 marks)	

Notes

B1: Selects the method of making a connection between x and w by writing $x=\dfrac{w+2}{3}$

Condone the use of a different letter than w

M1: Applies the process of substituting $x=\dfrac{w+2}{3}$ into $9x^3-5x^2+4x+7=0$

dM1: Depends on the previous M mark. Manipulates their equation into the form $aw^3+bw^2+cw+d\,(=0)$. Condone the use of a different letter then w consistent with B1 mark.

A1: At least two of a, b, c, d correct

A1: Fully correct equation, must be in terms of w

Alternative:

B1: Selects the method of giving three correct equations containing α, β and γ

M1: Applies the process of finding the new sum, new pair sum, new product

dM1: Depends on the previous M mark. Applies $w^3-(\text{new sum})w^2+(\text{new pair sum})w-(\text{new product})(=0)$ condone the use of any letter here.

A1: At least two of a, b, c, d correct

A1: Fully correct equation in term of w

Question	Scheme	Marks	AOs
7(a)(i)	$2 - \mathrm{i}$	B1	1.2
(ii)	Roots of polynomials with real coefficients occur in conjugate pairs, β and γ form a conjugate pair, α is real so δ must also be real. or Quartics have either 4 real roots, 2 real roots and 2 complex roots or 4 complex roots. As 2 complex roots and 1 real root therefore so δ must also be real. or As α real and only one root δ remaining, if complex it would need to have a complex conjugate, which it can't have so must be real	B1	2.4
		(2)	
(b)	$\alpha+\beta+\gamma+\delta=6$ $\Rightarrow 3+2+\mathrm{i}+2-\mathrm{i}+\delta=6 \Rightarrow \delta=...$	M1	3.1a
	$\delta=-1$	A1	1.1b
		(2)	
(c)	$\mathrm{f}(z)=(z-3)(z+1)(z-(2+\mathrm{i}))(z-(2-\mathrm{i}))=...$ Alternative pair sum $=(3)(2+\mathrm{i})+(3)(2-\mathrm{i})+(3)(-1)+(-1)(2+\mathrm{i})$ $+(-1)(2-\mathrm{i})+(2+\mathrm{i})(2-\mathrm{i})=...\{10\}$ triple sum $=(3)(2+\mathrm{i})(2-\mathrm{i})+(3)(-1)(2+\mathrm{i})$ $+(3)(-1)(2-\mathrm{i})+(-1)(2+\mathrm{i})(2-\mathrm{i})=...\{-2\}$ product $=(3)(2+\mathrm{i})(2-\mathrm{i})(-1)=...\{-15\}$	M1	3.1a
	$=(z^2-2z-3)(z^2-4z+5)$ $=z^4-6z^3+10z^2+2z-15$ $p=10, q=2, r=-15$	A1 A1	1.1b 1.1b
		(3)	
(d)	$z=\frac{1}{2},-\frac{3}{2}$	B1ft	1.1b
	$z=-1\pm\frac{\mathrm{i}}{2}$	B1ft	1.1b
		(2)	
			(9 marks)

Notes

(a)(i)
B1: Correct complex number
(a)(ii)
B1: Correct explanation.
(b)
M1: Uses 2 ± i and 1 together with the sum of roots = ± 6 to find a value for δ
A1: Correct value
(c)

4(i)	$\sum \alpha_i = -\frac{5}{3}$ and $\sum \alpha_i\alpha_j = 0$ This mark can be awarded if seen in part (ii) or part (iii)	B1	3.1a
	So $\alpha^2+\beta^2+\gamma^2+\delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2\left(\sum \alpha_i\alpha_j\right) = \ldots$	M1	1.1b
	$= \frac{25}{9} - 2\times 0 = \frac{25}{9}$	A1	1.1b
		(3)	
(ii)	$\sum \alpha_i\alpha_j\alpha_k = \frac{7}{3}$ and $\prod \alpha_i = 2$ or for $x = \frac{2}{w}$ used in equation This mark can be awarded if seen in part (i) or part (iii)	B1	2.2a
	So $2\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}\right) = 2\times\frac{\sum \alpha_i\alpha_j\alpha_k}{\alpha\beta\gamma\delta} = 2\times\frac{'\frac{7}{3}'}{'\frac{6}{3}'}$ or for $3\left(\frac{16}{w^4}\right)+5\left(\frac{8}{w^3}\right)-7\left(\frac{2}{w}\right)+6=0 \Rightarrow 6w^4-14w^3+\ldots=0$ leading to $\frac{14}{6}$	M1	1.1b
	$\left(=2\times\frac{7/3}{2}\right)\left(=\frac{14}{6}\right)=\frac{7}{3}$	A1	1.1b
		(3)	
(iii)	$(3-\alpha)(3-\beta)(3-\gamma)(3-\delta) = \ldots$ expands all four brackets Or equation with these roots is $3(3-x)^4+5(3-x)^3-7(3-x)+6=0$	M1	3.1a
	$= 81 - 27\left(\sum \alpha_i\right)+9\left(\sum \alpha_i\alpha_j\right)-3\left(\sum \alpha_i\alpha_j\alpha_k\right)+\prod \alpha_i$ $= 81-27\left(-\frac{5}{3}\right)+9(0)-3\left(\frac{7}{3}\right)+2$ Or expands to fourth power and constant terms and attempts product of roots $3x^4+\ldots+3\times3^4+5\times3^3-7\times3+6 \to \prod \alpha_i = \frac{"363"}{3}$	dM1	1.1b
	$=121$	A1	1.1b
		(3)	
			(9 marks)

Notes:

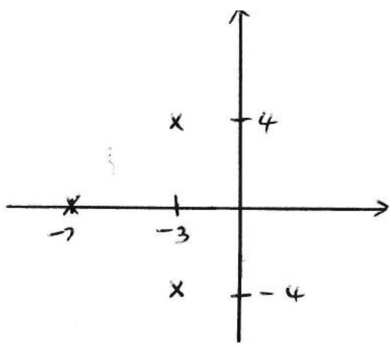
(i)

B1: Correct sum and pair sum of roots seen or implied. Must realise the pair sum is zero.
Note: These values can be seen anywhere in the candidate's solution

M1: Uses correct expression for the sum of squares.

A1: $\frac{25}{9}$. Allow this mark from incorrect sign on sum of squares (but they will score B0 if the sign is incorrect).

(ii)

Question	Scheme	Marks	AOs
2(a)	$z^* = -3-4\text{i}$ $(z-(-3+4\text{i}))(z-(-3-4\text{i})) = z^2+pz+q$ $\{\text{f}(z)=\}(z^2+pz+q)(z+r)$	M1	3.1a
	$(z^2+6z+25)(z+7)$	A1	1.1b
	Multiplies out $(z^2+6z+25)(z+7) = ...\alpha z^2+\beta z...$	M1	1.1b
	$z^3+13z^2+67z+175$ or $a=13, b=67$	A1	1.1b
		(4)	
	<u>Alternative 1</u> $z^* = -3-4\text{i}$ and uses product of roots = – 175 to find the third root	M1	3.1a
	Third root = –7	A1	1.1b
	Either Uses sum roots = – a to find a value for a **or** uses pair sum = b to find a value for b **Or** $(z-(-3+4\text{i}))(z-(-3-4\text{i}))(z-\text{their third root}) = ...$	M1	1.1b
	$a=13, b=67$	A1	1.1b
		(4)	
	<u>Alternative 2</u> $(-3+4\text{i})^3+a(-3+4\text{i})^2+b(-3+4\text{i})+175=0$ $\Rightarrow 117+44\text{i}+a(-7-24\text{i})+b(-3+4\text{i})+175=0$ Equates real and imaginary to form two linear simultaneous equations	M1	3.1a
	$117-7a-3b+175=0 \Rightarrow -7a-3b=-292$ $44-24a+4b=0 \Rightarrow -24a+4b=-44$	A1	1.1b
	Solves simultaneously to find values for a or b	M1	1.1b
	$a=13, b=67$	A1	1.1b
		(4)	
(b)	 $-3+4\text{i}, -3-4\text{i}$ -7	B1 B1	1.1b 2.2a
		(2)	

(c)	$-5+4\text{i},\ -5-4\text{i},\ -9$	B1ft	2.2a
		(1)	

(7 marks)

Notes:

(a)

M1: Uses the given root and its complex conjugate to form a quadratic equation. Uses the quadratic equation to write $\text{f}(z)$ in the form $(z^2+pz+q)(z+r)$ where *p*, *q* and *r* are real values

A1: Correct expression for $\text{f}(z)=(z^2+6z+25)(z+7)$

M1: Multiplies out and simplifies to find the z^2 or z term.

A1: Correct values for *a* and *b* or cubic

Alternative 1

M1: Uses the complex conjugate and product of roots = – 175 to find the third root.

A1: Correct third root

M1: A complete method to find the values of *a* or *b*. Either uses the sum and pairs sum or multiplies out three brackets $(z-(-3+4\text{i}))(z-(-3-4\text{i}))(z-\text{their third root})$ to find the z^2 or z term.

A1: Correct values for *a* and *b* or cubic

Alternative 2

M1: Substitutes $-3+4\text{i}$ or $-3-4\text{i}$ into $\text{f}(z)$, sets the real and imaginary parts = 0 to form two simultaneous equations in *a* and *b*.

A1: Correct, unsimplified equations.

M1: Solves simultaneous equations to find values for *a* or *b* following an attempt at $\text{f}(-3+4\text{i})=0$ or $\text{f}(-3-4\text{i})=0$. Allow this mark for seeing a value for *a* or *b* following simultaneous equation, you do not need to check.

A1: Correct values for *a* and *b*.

(b)

B1: Correctly plotting $-3+4\text{i},\ -3-4\text{i}$

B1: Correctly plotting – 7

(c)

B1ft: $-5+4\text{i},\ -5-4\text{i}$ and subtracts 2 from their real root shown on their Argand diagram

Question	Scheme	Marks	AOs
10(i)	$w = 3z - 1 \Rightarrow z = \dfrac{w+1}{3}$	B1	3.1a
	$\left(\dfrac{w+1}{3}\right)^4 + 5\left(\dfrac{w+1}{3}\right)^2 - 30 = 0$	M1	3.1a
	$\dfrac{1}{81}\left(w^4 + 4w^3 + 6w^2 + 4w + 1\right) + \dfrac{5}{9}\left(w^2 + 2w + 1\right) - 30 = 0$ leading to $w^4 + aw^3 + bw^2 + cw + d$ $\{= 0\}$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
	Alternative $p + q + r + s = 0,\quad pq + pr + ps + qr + qs + rs = 5$ $pqr + pqs + prs + qrs = 0,\quad pqrs = -30$	B1	3.1a
	New sum $= 3(p+q+r+s) - 4 = \ldots\{-4\}$ New pair sum $= 9(pq + pr + ps + qr + qs + rs) - 9(p+q+r+s) + 6 = \ldots\{51\}$ New triple sum $= 27(pqr + pqs + prs + qrs) - 18(pq + pr + ps + qr + qs + rs)$ $+6(p+q+r+s) - 4 = \ldots\{-94\}$ New product $= 81(pqrs) - 27(pqr + pqs + prs + qrs)$ $+9(pq + pr + ps + qr + qs + rs) - 3(p+q+r+s) + 1$ $= \ldots\{-2384\}$	M1	3.1a
	Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w$ $+ (\text{new product}) = 0$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	$\alpha + 2\alpha + \alpha - \beta = 0$ and $\alpha \times 2\alpha \times (\alpha - \beta) = -\dfrac{81}{4}$	M1 A1	3.1a 1.1b
	Solves simultaneously e.g. $4\alpha - \beta = 0 \Rightarrow \beta = 4\alpha$ $2\alpha^2(\alpha - 4\alpha) = -\dfrac{81}{4} \Rightarrow \alpha^3 = \dfrac{27}{8} \Rightarrow \alpha = \ldots$	M1	3.1a
	Uses their values $\alpha = \dfrac{3}{2}$ $\beta = 6$ to find the roots $\alpha, 2\alpha, \alpha - \beta$	M1	1.1b
	Roots 1.5, 3, – 4.5	A1	1.1b

		(5)	
(ii) (b)	$n=[(1.5\times3)+(1.5\times-4.5)+(3\times-4.5)]\times4$ Or Multiplies out $(x-3)\left(x-\frac{3}{2}\right)\left(x+\frac{9}{2}\right)$ or $(x-3)(2x-3)(2x+9)$ to achieve the form $4x^3+\ldots$	M1	1.1b
	$n=-63$ cso (must have correct roots in (a)	A1	1.1b
		(2)	
			(12 marks)

Notes:

(i)

B1: Selects the method of making a connection between *z* and *w* by writing $z=\frac{w+1}{3}$. Other variables may be used

M1: Applies the process of substituting their $z=\frac{w+1}{3}$ into $z^4+5z^2-30=0$

M1: Manipulates their equation into the form $w^4+aw^3+bw^2+cw+d=0$ having substituted their *z* in terms of *w*. Note that the "= 0" can be missing for this mark.

A1: At least two of *a*, *b*, *c*, *d* correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of *w*

(i) Alternative

B1: Selects the method of giving four correct equations containing *p*, *q*, *r* and *s*

M1: Applies the process of finding **at least three** of the new sum, new pair sum, new triple sum and new product. Condone slips but the intention is clear and uses their values.

M1: Applies $w^4-(\text{new sum})w^3+(\text{new pair sum})w^2-(\text{new triple sum})w+(\text{new product})=0$.

Condone use of any variable for this mark.

Note that the "= 0" can be missing for this mark.

A1: At least two of *a*, *b*, *c*, *d* correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of *w*

(ii) (a)

M1: Uses the sum and product to form two equations in α and β. Condone product = $\frac{81}{4}$ for this mark

Note: $4\alpha-\beta=-\frac{n}{4}$ or $4\alpha-\beta=81$ is M0

A1: Correct equations need not be simplied

M1: Solves simultaneous equations to find a value for α or β

M1: Uses their values for α and β to find the roots using α, 2α, $\alpha-\beta$. Condone third root as β

A1: Correct roots