4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

7. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50°) = 9 \tan (x - 25°)$$

(2)

12. (a) Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 4$$

giving any solutions to one decimal place.

(3)

12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ)$$

(5)

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

Leave blank

2. Solve, for $0 \leqslant x \leqslant 270°$, the equation

$$\frac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2$$

Give your answers in degrees to 2 decimal places.

(6)

Leave blank

8. (a) Prove that

$$2\text{cosec } 2A - \cot A \equiv \tan A, \qquad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$

(i) $2\text{cosec } 4\theta - \cot 2\theta = \sqrt{3}$

(ii) $\tan \theta + \cot \theta = 5$

Give your answers to 3 significant figures.

(6)

Leave blank

2. Solve, for $0 \leqslant \theta < 2\pi$,

$$2\cos 2\theta = 5 - 13\sin\theta$$

Give your answers in radians to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

7. (a) Given that

$$2\cos(x + 30)° = \sin(x - 30)°$$

without using a calculator, show that

$$\tan x° = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180$,

$$2\cos(2\theta + 40)° = \sin(2\theta - 20)°$$

Give your answers to one decimal place.

(4)

Leave blank

7. (a) Use the substitution $t = \tan x$ to show that the equation

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

can be written in the form

$$3t^4 + 8t^2 - 3 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 2\pi$,

$$4\tan 2x - 3\cot x \sec^2 x = 0$$

Give each answer in terms of π. You must make your method clear.

(4)

Leave blank

2. (a) Show that

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

may be expressed in the form $\operatorname{cosec}^2 x - \operatorname{cosec} x + k = 0$, where k is a constant.

(1)

(b) Hence solve for $0 \leqslant x < 360°$

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

Give each solution in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

8. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 \leqslant \theta < \frac{\pi}{2}$

(i) $\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta$

(ii) $\tan(\theta + 1)\cos(2\theta + 2) - \sin(2\theta + 2) = 2$

Give your answers in radians to 3 significant figures, as appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Leave blank

8. (a) Using the trigonometric identity for $\tan(A + B)$, prove that

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}, \qquad x \neq (2n + 1)30°, \qquad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-30° < x < 30°$,

$$\tan 3x = 11 \tan x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

7. (a) Prove that

$$\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x, \qquad x \neq (2n + 1)90°, \, n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve, for $-90° < \theta < 90°$,

$$\frac{2 - 2\cos 2\theta}{1 + \cos 2\theta} - 2 = 7 \sec \theta$$

Give your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

4. (a) Prove that

$$\frac{1-\cos 2x}{\sin 2x} \equiv \tan x, \qquad x \neq \frac{n\pi}{2}$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$3\sec^2\theta - 7 = \frac{1-\cos 2\theta}{\sin 2\theta}$$

Give your answers in radians to 3 decimal places, as appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

7. (a) For $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(2)

(b) Find the exact value of x for which

$$3g(x + 1) + \pi = 0$$

(3)

Leave blank

8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0\leqslant\theta\leqslant\pi$,

$$1+\sqrt{3}\tan\theta=\left(\sqrt{3}-\tan\theta\right)\tan\left(\pi-\theta\right)$$

Give your answers as multiples of π.

(6)

Answer ALL questions. Write your answers in the spaces provided.

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

2.

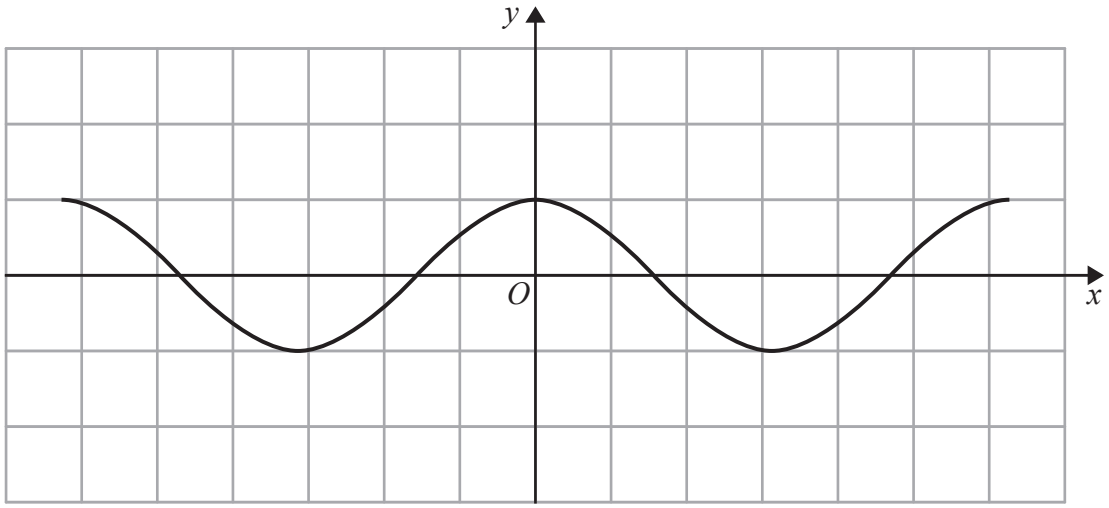

Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)