

Question	Scheme	Marks	AOs
11(a)			
	∧ shape in any position	B1	1.1b
	Correct $x$ -intercepts or coordinates	B1	1.1b
	Correct $y$ -intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a ∧ shape	B1	1.1b
	<b>(4)</b>		
(b)	$x = k$	B1	2.2a
	$k - (2x - 3k) = x - k \Rightarrow x = \dots$	M1	3.1a
	$x = \frac{5k}{3}$	A1	1.1b
	<b>Set notation is required here for this mark</b> $\left\{x : x < \frac{5k}{3}\right\} \cap \{x : x > k\}$	A1	2.5
	<b>(4)</b>		
(c)	$x = 3k$ <b>or</b> $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ <b>and</b> $y = 3 - 5k$	B1ft	2.2a
	<b>(2)</b>		

**(10 marks)****Notes****(a) Note that the sketch may be seen on Figure 4**

B1: See scheme

B1: Correct  $x$ -intercepts. Allow as shown or written as  $(k, 0)$  and  $(2k, 0)$  and condone coordinates written as  $(0, k)$  and  $(0, 2k)$  as long as they are in the correct places.B1: Correct  $y$ -intercept. Allow as shown or written as  $(0, -2k)$  or  $(-2k, 0)$  as long as it is in the correct place. Condone  $k - 3k$  for  $-2k$ .

B1: Correct coordinates as shown

**Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as  $y = 0, x = k$  etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.**

(b)

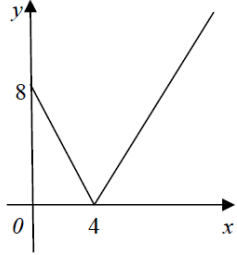
B1: Deduces the correct critical value of  $x = k$ . May be implied by e.g.  $x > k$  or  $x < k$ M1: Attempts to solve  $k - (2x - 3k) = x - k$  or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching  $k = \dots$  or  $x = \dots$  as long as they are solving the required equation.

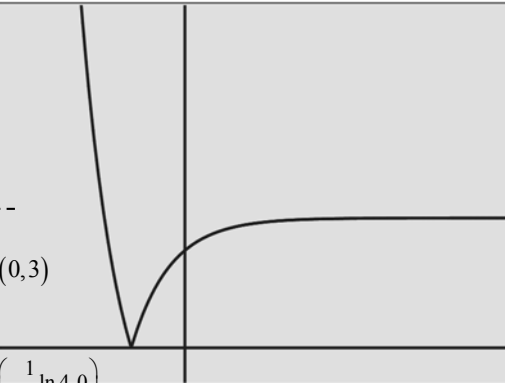
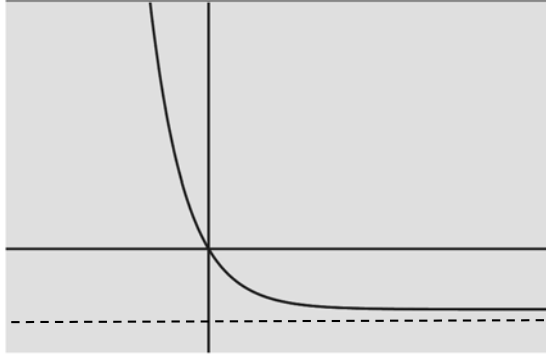
A1: Correct value

A1: Correct answer using the correct set notation.

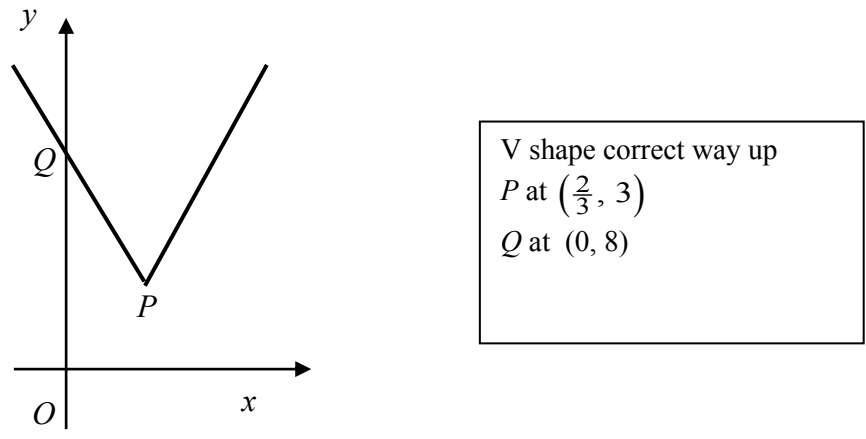
Question	Scheme	Marks	AOs
11(a)	$x = -4$ or $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x + 4) - 5 \Rightarrow x = \dots$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leq 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
			(7 marks)

Question Number	Scheme	Marks
<b>4(a)</b>	$fg(1) = f(2) = 7$	M1A1 <b>(2)</b>
<b>(b)</b>	Either $g(0)=3$ or $g(x \rightarrow \infty) \rightarrow 0.5$ $0.5 < g(x) \leq 3$	M1 A1 <b>(2)</b>
<b>(c)</b>	Attempt change of subject of $y = \frac{x+9}{2x+3} \Rightarrow y(2x+3) = x+9$ $\Rightarrow 2xy - x = 9 - 3y$ $\Rightarrow x(2y-1) = 9 - 3y \Rightarrow x = \frac{9-3y}{2y-1}$ $g^{-1}(x) = \frac{9-3x}{2x-1}, \quad 0.5 < x \leq 3$	M1 dM1 A1, B1 ft <b>(4)</b>
<b>(d)</b>	Attempts $f(0) = 2 \times 3 + 5 = 11 \Rightarrow k \leq 11$ Or $f(3) = 2 \times 0 + 5 = 5 \Rightarrow k > 5$ $5 < k \leq 11$	M1A1 A1 <b>(3)</b>
		<b>(11 marks)</b>

Question Number	Scheme	Marks
3(a)	 <p data-bbox="727 322 1278 360">V shape just in Quad 1 and correct position</p> <p data-bbox="946 398 1278 436">Meets/cuts <math>y</math> axis at <math>(0,8)</math></p> <p data-bbox="1015 472 1278 510">Meets <math>x</math> axis at <math>(4,0)</math></p>	<p data-bbox="1305 322 1342 360">B1</p> <p data-bbox="1305 398 1342 436">B1</p> <p data-bbox="1305 472 1342 510">B1</p> <p data-bbox="1465 544 1501 582">(3)</p>
(b)	<p data-bbox="531 618 600 656"><math>x = 1</math></p> <p data-bbox="523 689 863 728"><math>x + 5 = -(8 - 2x) \Rightarrow x = 13</math></p>	<p data-bbox="1305 618 1342 656">B1</p> <p data-bbox="1305 689 1390 728">M1A1</p> <p data-bbox="1465 730 1501 768">(3)</p>
(c)	<p data-bbox="563 801 788 840"><math>fg(5) = f(2) = -1</math></p>	<p data-bbox="1305 801 1390 840">M1A1</p> <p data-bbox="1465 875 1501 913">(2)</p>
(d)	<p data-bbox="347 958 900 1021"><math>f'(x) = 2x - 3 \Rightarrow \text{min at } x = \frac{3}{2} \Rightarrow \text{min} = -\frac{5}{4}</math></p> <p data-bbox="400 1025 663 1064">Maximum value = 5</p> <p data-bbox="411 1104 576 1167"><math>-\frac{5}{4}</math>, <math>f(x)</math>, 5</p>	<p data-bbox="1305 969 1390 1008">M1A1</p> <p data-bbox="1305 1025 1342 1064">B1</p> <p data-bbox="1305 1120 1342 1158">A1</p> <p data-bbox="1465 1178 1501 1216">(4)</p> <p data-bbox="1305 1249 1453 1288"><b>(12 marks)</b></p>

Question Number	Scheme	Marks
<p>11(a)</p>	 <p>Shape</p> <p>Asymptote <math>y = 4</math></p> <p><math>y</math> intercept <math>(0, 3)</math></p> <p>Touches <math>x</math> axis at <math>\left(-\frac{1}{3} \ln 4, 0\right)</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[4]</p>
<p>(b)</p>	 <p>Shape</p> <p>Asymptote <math>y = -2</math></p> <p>Passes through origin</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>
<p>(c)</p>	<p><math>f(x) &gt; -4</math></p>	<p>B1</p> <p>[1]</p>
<p>(d)</p>	<p><math>y = e^{-3x} - 4 \Rightarrow e^{-3x} = y + 4</math></p> <p><math>\Rightarrow -3x = \ln(y + 4)</math> and <math>x =</math></p> <p><math>f^{-1}(x) = -\frac{1}{3} \ln(x + 4)</math> or <math>\ln \frac{1}{(x+4)^{\frac{1}{3}}}</math>, <math>(x &gt; -4)</math> cao</p>	<p>M1</p> <p>dM1</p> <p>A1</p> <p>[3]</p>
<p>(e)</p>	<p><math>fg(x) = e^{-3 \ln\left(\frac{1}{x+2}\right)} - 4</math></p> <p><math>= (x + 2)^3 - 4</math>, <math>= x^3 + 6x^2 + 12x + 4</math></p>	<p>M1</p> <p>dM1, A1</p> <p>[3]</p> <p><b>(14 marks)</b></p>

Question Number	Scheme	Notes	Marks
6(a)(i)		V shape with vertex on $x$ -axis but <b>not</b> at the origin.	B1
		Correct V shape with $(0, a)$ or just $a$ and $(a, 0)$ or just $a$ marked in the correct places. Left branch must cross or touch the $y$ -axis. Allow coordinates the wrong way round if marked in the correct place.	B1
<b>(2)</b>			
(a)(ii)		Their part (i) translated down (by any amount) but clearly not left or right, or the correct shape i.e. a V with the vertex in 4 <sup>th</sup> quadrant.	B1ft
		A $y$ -intercept of $a - b$ on the positive $y$ -axis or intercepts of $a - b$ and $a + b$ on the positive $x$ -axis with $a + b$ to the right of $a - b$	B1
		A fully correct diagram.	B1
<b>(3)</b>			
(b)	$x - a - b = \frac{1}{2}x \Rightarrow x = \dots$ <p style="text-align: center;"><b>or</b></p> $-x + a - b = \frac{1}{2}x \Rightarrow x = \dots$	Solves $x - a - b = \frac{1}{2}x$ <b>or</b> solves $-x + a - b = \frac{1}{2}x$ as far as $x = \dots$ Allow $<$ or $>$ for $=$ .	M1
	$x - a - b = \frac{1}{2}x \Rightarrow x = \dots$ <p style="text-align: center;"><b>and</b></p> $-x + a - b = \frac{1}{2}x \Rightarrow x = \dots$	Solves $x - a - b = \frac{1}{2}x$ <b>and</b> solves $-x + a - b = \frac{1}{2}x$ as far as $x = \dots$ Allow $<$ or $>$ for $=$ .	M1
	$\frac{2}{3}(a - b) < x < 2(a + b)$	ddM1: Chooses inside region. A1: Allow alternatives e.g. $x < 2(a + b)$ <b>and</b> $x > \frac{2}{3}(a - b)$ , $x < 2(a + b) \cap x > \frac{2}{3}(a - b)$ , $\left(\frac{2}{3}(a - b), 2(a + b)\right)$ but not $x < 2(a + b), x > \frac{2}{3}(a - b)$	ddM1A1
<b>(4)</b>			
<b>(9 marks)</b>			

Qu	Scheme	Marks
7 (a)(i)	Substitute (0, 5) to give $ b  = 5$ so $b = \pm 5$ Substitute $(\frac{1}{3}, 0)$ to give $ \frac{1}{3}a + b  = 0$ so $a = \mp 15$	B1 M1 A1
(ii)	Gives equation as $y =  -15x + 5 $ or $y =  15x - 5 $	B1 (4)
(b)		B1 B1 B1 (3)
		(7 marks)

(a)

(i)

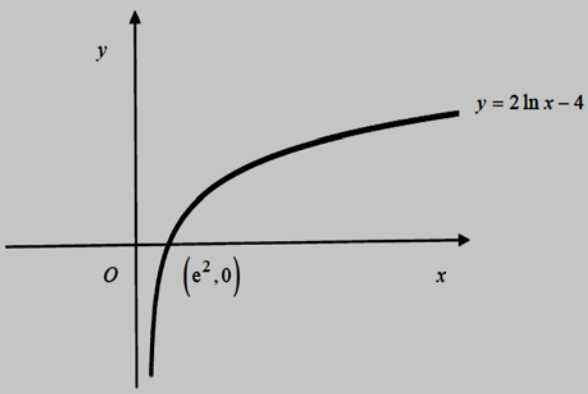
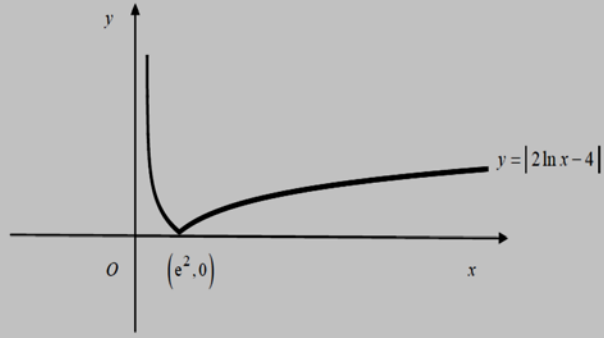
**B1:** For both  $b = \pm 5$  not just  $|b| = 5$ **M1:** Substitute  $(\frac{1}{3}, 0)$  to give  $|\frac{1}{3}a + b| = 0$ . This mark is implied by  $a = \pm 3 \times$  value of  $b$ **A1:**  $a = 15$  corresponding to  $b = -5$  and  $a = -15$  corresponding to  $b = 5$ If they write down an equation rather than giving values of  $a$  and  $b$  thenJust  $y = |-15x + 5|$  or  $y = |15x - 5|$  scores B0, M1, A0Both  $y = |-15x + 5|$  and  $y = |15x - 5|$  scores B1, M1, A1Linear equations  $y = -15x + 5$  and/or  $y = 15x - 5$  (without the modulus) only score B0 M1 A0

(ii) Note that this is an A1 mark on e-pen

**B1:**  $y = |-15x + 5|$  or  $y = |15x - 5|$  or allow equations such as for this mark only  $f(x) = \begin{cases} 15x - 5 & x \geq \frac{1}{3} \\ -15x + 5 & x < \frac{1}{3} \end{cases}$ If candidates don't state (i), (ii) and write down just  $y = |-15x + 5|$  they would score (i) B0 M1 A0 (ii) B1

(b) There must be a sketch to score any of these marks.

**B1:** V shape the correct way up any position but not on the  $x$ -axis. Accept V's that don't have symmetry**B1:**  $P$  at  $(\frac{2}{3}, 3)$  Score if the coordinates are stated within the text OR marked on the axes. If they appear in both then the graph takes precedence.**B1:** For **crossing** the  $y$ -axis at (0, 8). Accept 8 marked on the correct axis. Condone (8,0) marked on the correct axis

Question Number	Scheme		Marks
9(a)(i)		Shape	B1
		$(e^2, 0)$	B1
		Asymptote $x = 0$	B1
<b>(3)</b>			
(a)(ii)		Shape	B1ft
		Asymptote and coordinate	B1ft
<b>(2)</b>			
(b)	$2 \ln x - 4 = 4 \Rightarrow \ln x = 4 \Rightarrow x = e^4$		M1A1
	$2 \ln x - 4 = -4 \Rightarrow \ln x = 0 \Rightarrow x = 1$		M1A1
<b>(4)</b>			
(c)	$gf(x) = e^{2 \ln x - 4 + 5} - 2 = e^1 \times e^{2 \ln x} - 2 = ex^2 - 2$		M1,dM1A1
			<b>(3)</b>
(d)	$gf(x) > -2$		B1
			<b>(1)</b>
			<b>(13 marks)</b>

(a)(i)

B1: For a logarithmic shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1: Intersection with the  $x$  axis at  $(e^2, 0)$ .

Allow  $e^2$  marked on the  $x$  axis. Condone  $(0, e^2)$  being marked on the positive  $x$  axis.

Do not allow  $e^2$  appearing as 7.39 for this mark unless  $e^2$  is seen in the body of the script.

Allow if the coordinate is given in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then the ones on the curve take precedence.

B1: **Equation** of asymptote is  $x = 0$  (do not allow “ $y$ -axis”). Note that the curve must appear to have an asymptote at  $x = 0$

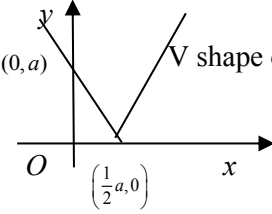
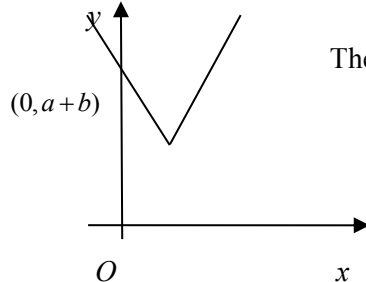
(a)(ii)

B1ft: For either the correct shape or a reflection of their “negative” curve in (a) in the  $x$ -axis. For this to be scored it must have appeared both above and below the  $x$ -axis. The curve to the lhs of the intercept must appear to have the correct curvature

B1ft: Score for the correct coordinates and asymptote. Alternatively follow through on the coordinates and asymptote given in part (a) as long as the curve appeared both above and below the  $x$ -axis and the curve approaches the same asymptote stated in (a)(i). Do not penalise “ $y$ -axis” given as the asymptote twice – i.e. penalise in (a)(i) only.

**If the curves are sketched on the same axes – it must be clear which curve is which – if in doubt use review.**



Question Number	Scheme	Marks
<p><b>6.(a)(i)</b></p>	 <p>V shape on <math>x</math>-axis <b>or</b> coordinates <math>(\frac{1}{2}a, 0)</math> <b>and</b> <math>(0, a)</math></p> <p>Correct shape, position and coordinates</p>	<p>B1</p> <p>B1</p>
<p><b>(ii)</b></p>	 <p>Their "V" shape translated up or <math>(0, a+b)</math></p> <p>Correct shape, position and <math>(0, a+b)</math></p>	<p>B1ft</p> <p>B1</p> <p><b>(4)</b></p>
<p><b>(b)</b></p>	<p>States or uses <math>a + b = 8</math></p> <p>Attempts to solve <math> 2x - a  + b = \frac{3}{2}x + 8</math> in either <math>x</math> or with <math>x = c</math></p> $2c - a + b = \frac{3}{2}c + 8 \Rightarrow kc = f(a, b)$ <p>Combines <math>kc = f(a, b)</math> with <math>a + b = 8 \Rightarrow c = 4a</math></p>	<p>B1</p> <p>M1</p> <p>dM1 A1</p> <p><b>(4)</b></p> <p><b>(8 marks)</b></p>

(a)(i)

B1 V shape sitting anywhere on the  $x$ -axis **or** for  $(\frac{1}{2}a, 0)$  and  $(0, a)$  lying on the curve.

Condone non-symmetrical graphs and ones lying on just one side of the  $y$ -axis

B1 V shape sitting on the positive  $x$ -axis at  $(\frac{1}{2}a, 0)$ , cutting the  $y$ -axis at  $(0, a)$  and lying in both quadrants 1 and 2

Accept  $\frac{1}{2}a$  and  $a$  marked on the correct axis. Condone say  $(a, 0)$  for  $(0, a)$  as long as it is on the correct axis.

Condone a dotted line appearing on the diagram as many reflect  $y = 2x - a$  to sketch  $y = |2x - a|$

If it is a solid line then it would not score the shape mark.

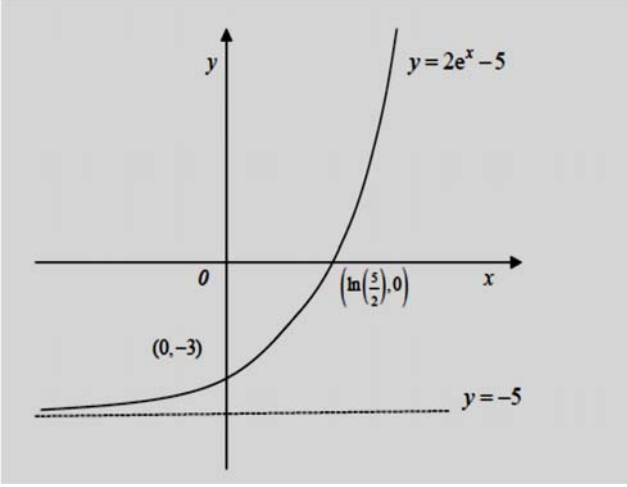
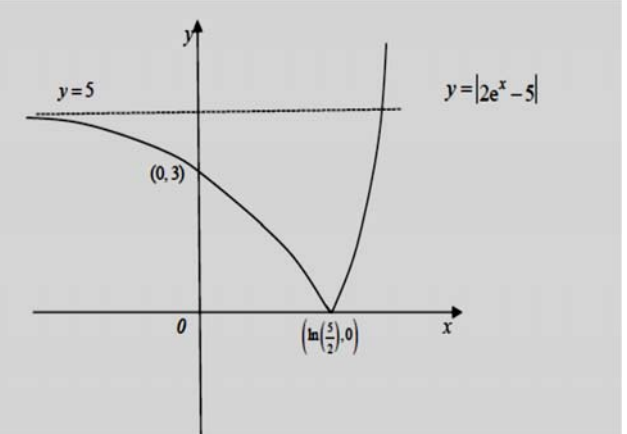
(a)(ii)

B1ft Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.

Alternatively score for the  $(0, a+b)$  lying on the curve

B1 V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the  $y$ -axis at  $(0, a+b)$

Ignore any coordinates given for the vertex.

Question Number	Scheme	Marks
2.(ai)	 <p>A Cartesian coordinate system showing the graph of the function <math>y = 2e^x - 5</math>. The x-axis and y-axis are shown, with the origin labeled '0'. The graph is an increasing curve that passes through the point <math>(\ln(\frac{5}{2}), 0)</math> on the x-axis and <math>(0, -3)</math> on the y-axis. A horizontal dashed line represents the asymptote <math>y = -5</math>.</p>	<p>Shape B1</p> <p><math>(\ln(\frac{5}{2}), 0)</math> and <math>(0, -3)</math> B1</p> <p><math>y = -5</math> B1</p> <p style="text-align: right;"><b>(3)</b></p>
(aii)	 <p>A Cartesian coordinate system showing the graph of the function <math>y =  2e^x - 5 </math>. The x-axis and y-axis are shown, with the origin labeled '0'. The graph is a V-shaped curve with its vertex at <math>(\ln(\frac{5}{2}), 0)</math> on the x-axis. It passes through the point <math>(0, 3)</math> on the y-axis. A horizontal dashed line represents the asymptote <math>y = 5</math>.</p>	<p>Shape inc cusp B1ft</p> <p><math>(\ln(\frac{5}{2}), 0)</math> and <math>(0, 3)</math> B1ft</p> <p><math>y = 5</math> B1ft</p> <p style="text-align: right;"><b>(3)</b></p>
(b)	<p><math>x \geq \ln\left(\frac{5}{2}\right)</math></p>	<p>B1 ft</p> <p style="text-align: right;"><b>(1)</b></p>
(c)	<p><math>2e^x - 5 = -2 \Rightarrow (x) = \ln\left(\frac{3}{2}\right)</math></p> <p><math>(x) = \ln\left(\frac{7}{2}\right)</math></p>	<p>M1A1</p> <p>B1</p> <p style="text-align: right;"><b>(3)</b></p> <p style="text-align: right;"><b>(10 marks)</b></p>

Question Number	Scheme	Marks
5. (a)	<p>V shaped graph</p> <p>Touches <math>x</math> axis at <math>\frac{3}{4}</math> and cuts <math>y</math> axis at 3</p>	<p>B1</p> <p>B1</p> <p>(2)</p>
(b)	<p>Solves <math>4x - 3 = 2 - 2x</math> or <math>3 - 4x = 2 - 2x</math> to give either value of <math>x</math></p> <p>Both <math>x = \frac{5}{6}</math> and <math>x = \frac{1}{2}</math></p> <p>or <math>x &gt; \frac{5}{6}</math> or <math>x &lt; \frac{1}{2}</math></p>	<p>M1</p> <p>A1</p> <p>dM1A1</p> <p>(4)</p>
(c)	<p>Draws graph Or solves <math> 4x - 3  = 1\frac{1}{2} - 2x</math> to give one soln <math>x = \frac{3}{4}</math></p> <p>Accept for all values of <math>x</math> except <math>x = \frac{3}{4}</math> Or <math>(x \in \mathbb{R},) x \neq \frac{3}{4},</math> or <math>x &lt; \frac{3}{4}, x &gt; \frac{3}{4}</math></p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>(8 marks)</p>

(a)

B1 A 'V' shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a 'dotted' extension of  $y = 4x - 3$  appearing under the  $x$  axis.

B1 The graph **meets** the  $x$  axis at  $x = \frac{3}{4}$  and **crosses** the  $y$  axis at  $y = 3$ . Do not allow multiple meets or crosses  
If they have lost the previous B1 mark for an extra section of graph underneath the  $x$  axis allow for **crossing** the  $x$  axis at  $x = \frac{3}{4}$  and **crosses** the  $y$  axis at  $y = 3$ .

Accept marked elsewhere on the page with  $A$  and  $B$  marked on the graph and  $A = \left(\frac{3}{4}, 0\right)$  and  $B = (0, 3)$

Condone  $\left(0, \frac{3}{4}\right)$  and  $(3, 0)$  marked on the correct axis

(b)

M1 Attempts to solve  $|4x - 3| \dots 2 - 2x$  finding at least one solution. You may see ... replaced by either  $=$  or  $>$

Accept as evidence  $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$

Accept as evidence  $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..$ , or  $x < ..$

A1 Both critical values  $x = \frac{5}{6}$  and  $x = \frac{1}{2}$ , or one inequality, accept  $x > \frac{5}{6}$  or  $x < \frac{1}{2}$

Accept  $x = 0.83$  and  $x = 0.5$  for the critical values

Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.

Eg if M1 has been scored for  $4x - 3 = 2 - 2x \Rightarrow x = 0.83$  and  $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$

A correct application of M1 would be  $x < -2.5, x > 0.83$

A1 Correct answer only  $x < \frac{1}{2}$  or  $x > \frac{5}{6}$ .

Accept  $x < 0.5, x > 0.83$

(c)

M1 **Either** sketch both lines showing a single intersection at the point  $x = \frac{3}{4}$

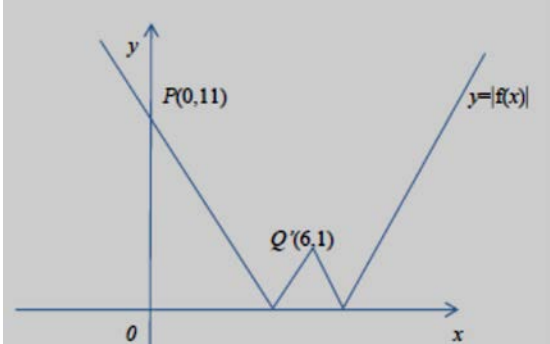
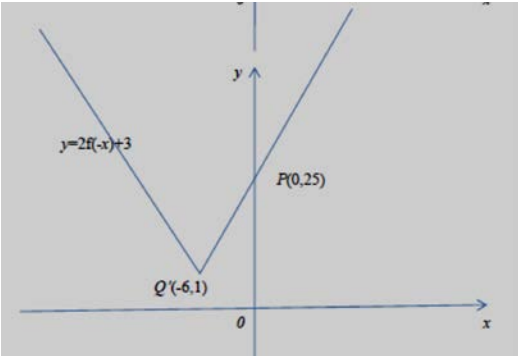
**Or** solves  $|4x - 3| = 1\frac{1}{2} - 2x$  using both  $4x - 3 = 1\frac{1}{2} - 2x$  and  $-4x + 3 = 1\frac{1}{2} - 2x$  **giving one solution**  $x = \frac{3}{4}$

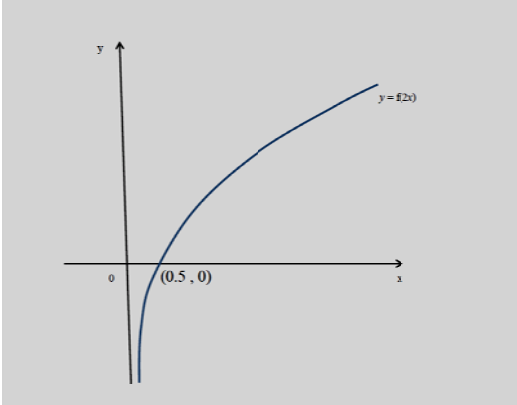
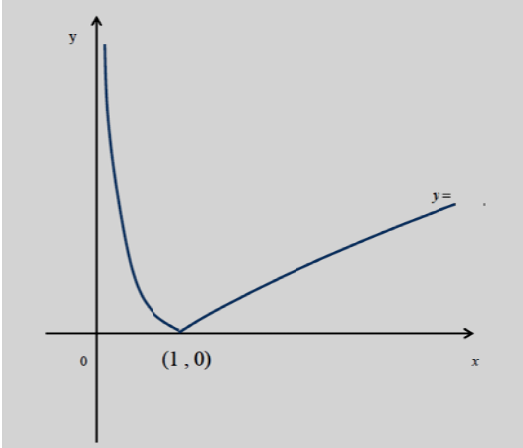
Accept  $|4x - 3| > 1\frac{1}{2} - 2x$  using both  $4x - 3 > 1\frac{1}{2} - 2x$  and  $-4x + 3 > 1\frac{1}{2} - 2x$  **giving one solution**  $x \dots \frac{3}{4}$

If two values are obtained using either method it is M0A0

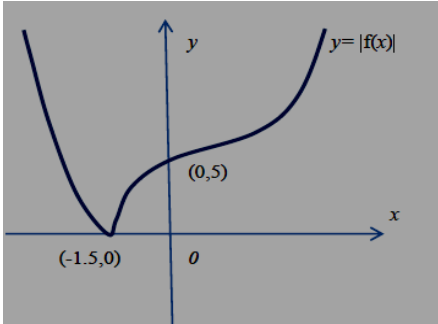
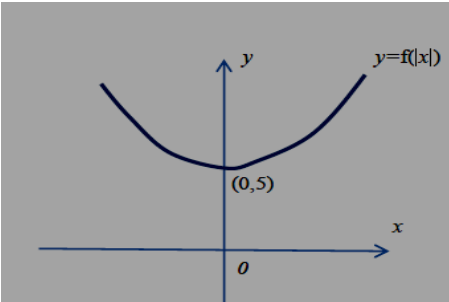
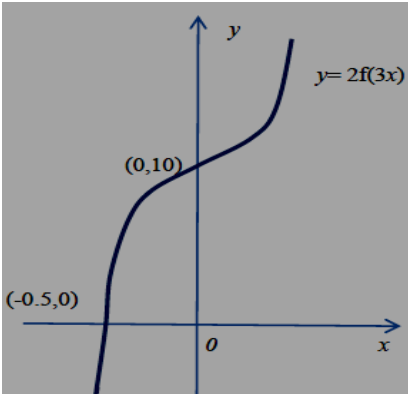
A1 States that the solution set is all values apart from  $x = \frac{3}{4}$ . Do not isw in this question. Score their final

statement. Accept versions of all values of  $x$  except  $x = \frac{3}{4}$  or  $x \in \mathbb{R}$ ,  $x \neq \frac{3}{4}$ , or  $x < \frac{3}{4}$ ,  $x > \frac{3}{4}$

Question Number	Scheme	Marks
<p>4.(a)</p>		<p>'W' Shape B1 (0, 11) and (6, 1) B1</p> <p>(2)</p>
<p>(b)</p>		<p>'V' shape B1 (-6,1) B1 (0,25) B1</p> <p>(3)</p>
<p>(c)</p>	<p>One of <math>a = 2</math> or <math>b = 6</math> <math>a = 2</math> and <math>b = 6</math></p>	<p>B1 B1</p> <p>(2)</p> <p><b>(7 marks)</b></p>

Question Number	Scheme	Marks
2.(a)		<p>Shape B1 (0.5, 0) B1</p> <p>(2)</p>
(b)		<p>Shape B1 (1,0) B1 Cusp at (1,0) B1</p> <p>(3)</p> <p>(5 marks)</p>

Question Number	Scheme	Marks
4.(a)	$f(x) \geq 3$	M1A1 (2)
(b)	An attempt to find $2 3-4x +3$ when $x=1$ Correct answer $fg(1)=5$	M1 A1 (2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\frac{3-y}{4}$ $g^{-1}(x)=\frac{3-x}{4}$	M1 A1 (2)
(d)	$[g(x)]^2 = (3-4x)^2$ $gg(x) = 3-4(3-4x)$ $gg(x) + [g(x)]^2 = 0 \Rightarrow -9+16x+9-24x+16x^2 = 0$ $16x^2 - 8x = 0$ $8x(2x-1) = 0 \Rightarrow x = 0, 0.5$ oe	B1 M1 A1 M1A1 (5) <b>(11 marks)</b>

Question Number	Scheme	Marks
4.(a)		<p>Shape including cusp B1</p> <p>(-1.5, 0) <b>and</b> (0, 5) B1</p> <p>(2)</p>
(b)		<p>Shape B1</p> <p>(0,5) B1</p> <p>(2)</p>
(c)		<p>Shape B1</p> <p>(0,10) B1</p> <p>(-0.5, 0) B1</p> <p>(3)</p> <p><b>(7 marks)</b></p>

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the  $x$ -axis at  $(-1.5, 0)$  **and** crossing the  $y$ -axis at  $(0,5)$

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the  $y$ - axis

B1  $(0,5)$  lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to  $f(x)$

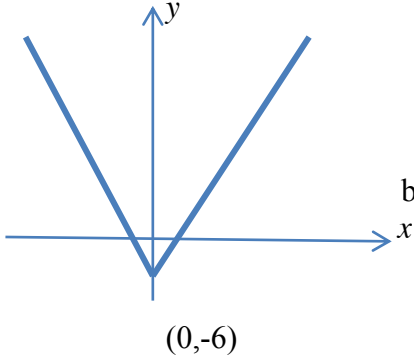
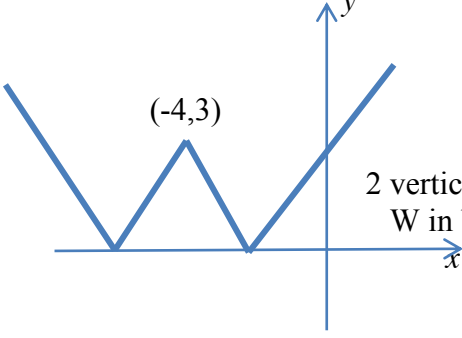
B1 Curve **crosses** the  $y$  axis at  $(0, 10)$ . The curve must appear in both quadrants 1 and 2

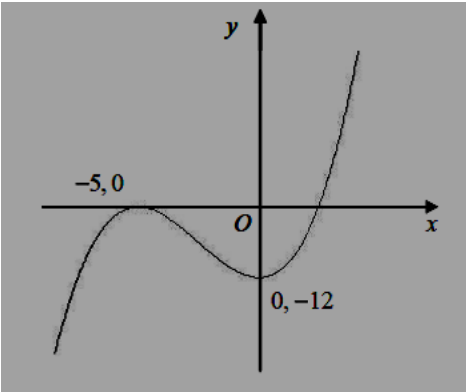
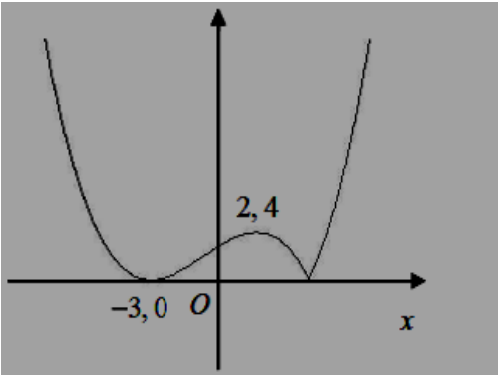
B1 Curve **crosses** the  $x$  axis at  $(-0.5, 0)$ . The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using  $(0,3)$  as an example, accept both  $(3,0)$  or 3 written on the  $y$  axis (as long as the curve passes through the point)

**Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1 Otherwise follow scheme**



Question Number	Scheme	Marks
3 (a)	 <p>V shape</p> <p>vertex on y axis &amp; both branches of graph cross x axis</p> <p>'y' co-ordinate of R is -6</p> <p>(0,-6)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(b)	 <p>W shape</p> <p>2 vertices on the negative x axis. W in both quad 1 &amp; quad 2.</p> <p><math>R' = (-4, 3)</math></p>	<p>B1</p> <p>B1dep</p> <p>B1</p> <p>(3)</p> <p>6 Marks</p>
4 (a)	$y = 4 - \ln(x + 2)$ $\ln(x + 2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ <p style="text-align: right;">oe</p>	<p>M1</p> <p>M1A1</p> <p>(3)</p>
(b)	$x \leq 4$	<p>B1</p> <p>(1)</p>
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$	<p>M1</p> <p>dM1A1</p> <p>(3)</p>
(d)	$fg(x) \leq 4$	<p>B1ft</p> <p>(1)</p> <p>8 Marks</p>

Question No	Scheme	Marks
2	<p>(a)</p>  <p>Shape B1 x coordinates correct B1 y coordinates correct B1</p> <p>(3)</p> <p>(b)</p>  <p>Shape B1 Max at (2,4) B1 Min at (-3,0) B1</p> <p>(3)</p> <p><b>6 marks</b></p>	

- (a)
- B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross  $x$  axis.
  - B1 The  $x$ - coordinates of  $P'$  and  $Q'$  are  $-5$  and  $0$  respectively. This is for translating the curve 2 units left. The minimum point  $Q'$  must be on the  $y$  axis. Accept if  $-5$  is marked on the  $x$  axis for  $P'$  with  $Q'$  on the  $y$  axis (marked  $-12$ ).
  - B1 The  $y$ - coordinates of  $P'$  and  $Q'$  are  $0$  and  $-12$  respectively. This is for the stretch  $\times 3$  parallel to the  $y$  axis. The maximum  $P'$  must be on the  $x$  axis. Accept if  $-12$  is marked on the  $y$  axis for  $Q'$  with  $P'$  on the  $x$  axis (marked  $-5$ )
- (b)
- B1 The curve below the  $x$  axis reflected in the  $x$  axis and the curve above the  $x$  axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the  $x$  axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.
  - B1 Both the  $x$ - and  $y$ - coordinates of  $Q'$ ,  $(2,4)$  given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum. Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.
  - B1 Both the  $x$ - and  $y$ - coordinates of  $P'$ ,  $(-3,0)$  given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept  $(0, -3)$  marked on the correct axis.

Question No	Scheme	Marks
3	(a) $20 \text{ (mm}^2\text{)}$	<p><b>B1</b></p> <p><b>M1</b></p> <p>(1)</p>

	(b) '40' = 20 $e^{1.5t}$ → $e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20} = (2)$  Correct order $1.5t = \ln'2'$ → $t = \frac{\ln c}{1.5}$ $t = \frac{\ln 2}{1.5} = (\text{awrt } 0.46)$  12.28 or 28 (minutes)	A1  M1 A1 A1  <b>(5)</b>  <b>(6 marks)</b>
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**(a)**

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

**(b)**

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to  $e^{1.5t} = \text{constant}$ . Accept non numerical answers.

A1  $e^{1.5t} = \frac{40}{20}$  or 2

M1 Correct ln work to find t. Eg  $e^{1.5t} = \text{constant} \rightarrow 1.5t = \ln(\text{constant}) \rightarrow t =$

The order must be correct. Accept non numerical answers. **See below for correct alternatives**

A1 Achieves either  $\frac{\ln(2)}{1.5}$  or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

#### Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to  $\ln('40') = \ln 20 + \ln e^{1.5t}$

A1  $\ln(40) = \ln 20 + 1.5t$

M1 Make t the subject with correct ln work.

$$\ln('40') - \ln 20 = 1.5t \text{ or } \ln\left(\frac{40}{20}\right) = 1.5t \rightarrow t =$$

A1,A1 are the same

#### Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into  $20e^{1.5t}$  to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into  $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader