Question	Scheme	Marks	AOs
15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of $n = 1, 2, 3$ and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say $n = 0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for $n=1, 2, 3$ and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and **states** even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

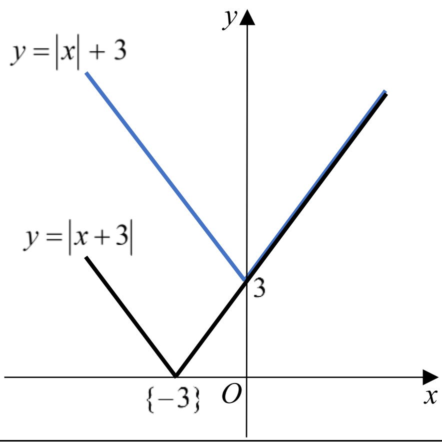
- all terms are even
- sight of a factorised expression E.g. $8p^3-12p^2+6p+4=2(4p^3-6p^2+3p+2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if m^3+5 is odd then m is even"
- "this is contradiction, so proven."
- "So if m^3+5 is odd them m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a counter example to the statement "if m^3+5 is odd then m must be even" such as when $m=\sqrt[3]{2}$ then they can score special case mark B1

Question	Scheme		Marks	AOs
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."			
(a)	E.g. $m = \sqrt{3}, n = \sqrt{12}$		M1	1.1b
	$\{mn =\}$ $(\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational		A1	2.4
			(2)	
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1	1.1b
		Superimposes the graph of $y = \|x+3\|$ on top of the graph of $y = \|x\| + 3$	M1	3.1a
	the graph of $y = \|x\| + 3$ is either the same or above the graph of $y = \|x+3\|$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \|x\| + 3$ is above the graph of $y = \|x+3\|$		A1	2.4
			(3)	
(b)(ii) Way 2	**<u>Reason 1</u>** When $x \geq 0$, $\|x\| + 3 = \|x+3\|$ **<u>Reason 2</u>** When $x < 0$, $\|x\| + 3 > \|x+3\|$	Any one of Reason 1 or Reason 2	M1	3.1a
		Both Reason 1 and Reason 2	A1	2.4
			(5 marks)	

Notes for Question 3

(a)	
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3}, \sqrt{12}$; $\sqrt{2}, \sqrt{8}$; $\sqrt{5}, -\sqrt{5}$; $\frac{1}{\pi}, 2\pi$; $3\mathrm{e}, \frac{4}{5\mathrm{e}}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3\mathrm{e})\left(\frac{4}{5\mathrm{e}}\right) = \frac{12}{5}$ $\Rightarrow$ untrue is sufficient for M1A1
(b)(i)	
B1:	See scheme
(b)(ii)	
M1:	For constructing a method of comparing $\|x\| + 3$ with $\|x+3\|$. See scheme.
A1:	Explains fully why $\|x\| + 3 \geq \|x+3\|$. See scheme.
Note:	Do not allow either $x > 0$, $\|x\| + 3 \geq \|x+3\|$ or $x \geq 0$, $\|x\| + 3 \geq \|x+3\|$ as a valid reason
Note	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \leq 0$, $\|x\| + 3 > \|x+3\|$ for A1

Question	Scheme	Marks	AOs
9	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h)-\cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
		(5 marks)	

Notes for Question 9

B1:	Gives the correct fraction such as $\frac{\cos(\theta+h)-\cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta)-\cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h)-\cos\theta}{(\theta+h)-\theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $\mathrm{f}'(\theta)$ where $\mathrm{f}(\theta) = \cos\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ where $y = \cos\theta$ used in place of $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)$

Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

(i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either *n* odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all *n*

Example of an algebraic proof

For $n=2m,\quad n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1,\quad n^2+2=(2m+1)^2+2=\ldots$ FYI $\left(4m^2+4m+3\right)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\frac{4m^2+2}{4} = m^2 + \frac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\frac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\frac{n^2+2}{4} = \frac{4m^2+4m+3}{4} = m^2+m+\frac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\frac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction 'Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ '	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2} \times \sqrt{2k-1}$

dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p, q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)