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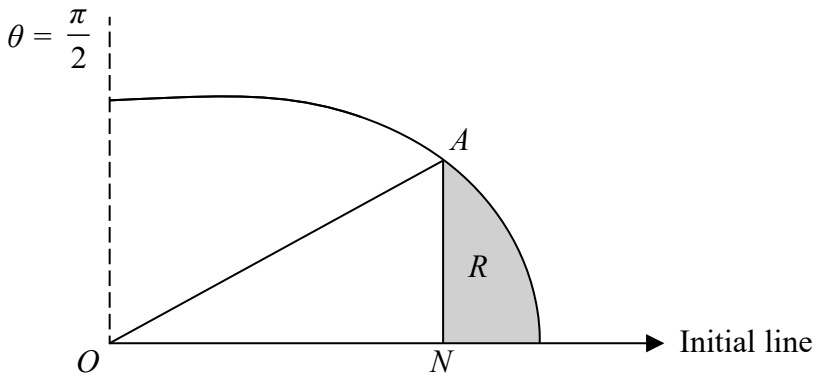

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(9)

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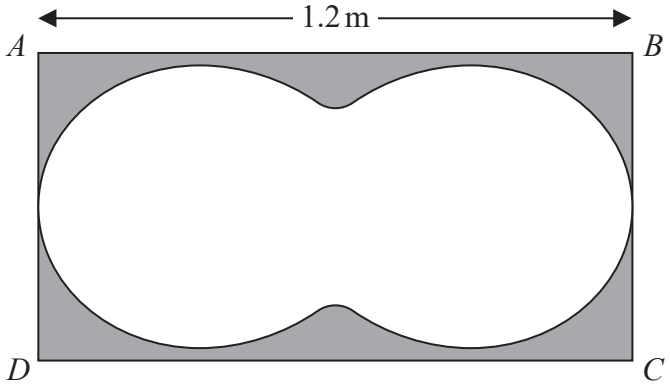

Diagram not to scale

Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle $ABCD$. The length of the table, AB, is 1.2 m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leqslant \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0.2$

(2)

Hence, given that $AD = 60$ cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.

(8)

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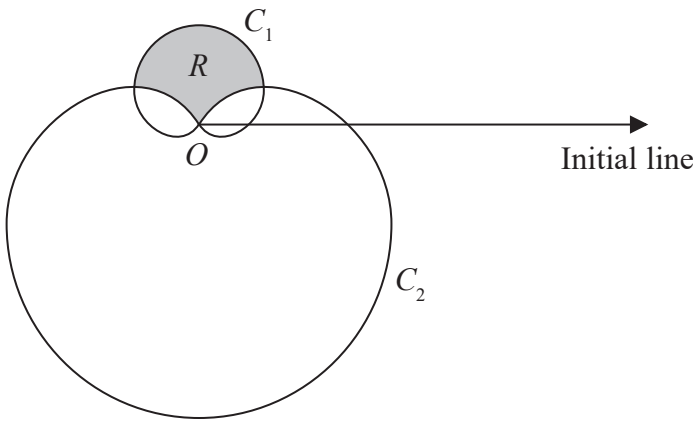

Figure 1

Figure 1 shows a sketch of two curves C_1 and C_2 with polar equations

$$C_1: r = (1 + \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

$$C_2: r = 3(1 - \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

The region R lies inside C_1 and outside C_2 and is shown shaded in Figure 1.

Show that the area of R is

$$p\sqrt{3} - q\pi$$

where p and q are integers to be determined.

(9)

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6. The curve C has equation

$$r = a(p + 2\cos\theta) \qquad 0 \leqslant \theta < 2\pi$$

where a and p are positive constants and $p > 2$

There are exactly four points on C where the tangent is perpendicular to the initial line.

(a) Show that the range of possible values for p is

$$2 < p < 4$$

(5)

(b) Sketch the curve with equation

$$r = a(3 + 2\cos\theta) \qquad 0 \leqslant \theta < 2\pi \qquad \text{where } a > 0$$

(1)

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve with equation

$$r = 20(3 + 2\cos\theta) \qquad 0 \leqslant \theta < 2\pi$$

where r is measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

(c) determine how long it will take to completely fill the pond with water using the hosepipe, according to the model. Give your answer to the nearest minute.

(7)

(d) State a limitation of the model.

(1)

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7.

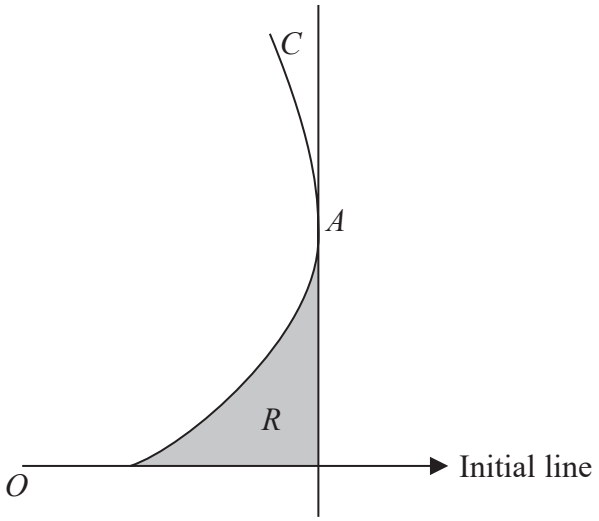

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$r = 1 + \tan\theta \qquad 0 \leqslant \theta < \frac{\pi}{3}$$

Figure 1 also shows the tangent to C at the point A.
This tangent is perpendicular to the initial line.

(a) Use differentiation to prove that the polar coordinates of A are $\left(2, \frac{\pi}{4}\right)$

(4)

The finite region R, shown shaded in Figure 1, is bounded by C, the tangent at A and the initial line.

(b) Use calculus to show that the exact area of R is $\frac{1}{2}(1 - \ln 2)$

(6)

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1.

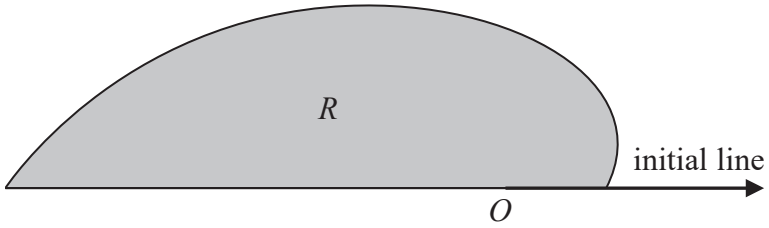

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = 2\sqrt{\sinh\theta + \cosh\theta} \qquad 0 \leqslant \theta \leqslant \pi$$

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line with equation $\theta = \pi$

Use algebraic integration to determine the exact area of R giving your answer in the form $pe^q - r$ where p, q and r are real numbers to be found.

(4)

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4. (a) Sketch the polar curve C, with equation

$$r = 3 + \sqrt{5}\cos\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

On your sketch clearly label the pole, the initial line and the value of r at the point where the curve intersects the initial line.

(2)

The tangent to C at the point A, where $0 < \theta < \frac{\pi}{2}$, is parallel to the initial line.

(b) Use calculus to show that at A

$$\cos\theta = \frac{1}{\sqrt{5}}$$

(4)

(c) Hence determine the value of r at A.

(1)

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