Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or $y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$	M1	1.1b
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0 \Rightarrow (2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow (r-9)^2-81+36=0 \Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Question Number	Scheme	Marks
15. (a)	gradient $=\frac{11-3}{6-0}, =\frac{4}{3}$	M1 A1 **[2]**
(b)	Mid-point of XY = (3, 7)	M1 A1
	ZM has gradient $-\frac{1}{m}$ $\left(=-\frac{3}{4}\right)$	B1ft
	Either : $y-"7"="-\frac{3}{4}"\left(x-"3"\right)$ **or**: $y="-\frac{3}{4}"x+c$ and $"7"="-\frac{3}{4}"\left("3"\right)+c \Rightarrow c="9\frac{1}{4}"$	M1
	$4y+3x-37=0$ or $y-7=-\frac{3}{4}\left(x-3\right)$ Or $y=-\frac{3}{4}x+9\frac{1}{4}$	A1 **[5]**
(c)	Substitute y = 10 into their line equation to give x =	M1
	x = -1	A1 **[2]**
(d)	$(r^2)=(-1-0)^2+(10-3)^2$ or $(r^2)=(-1-6)^2+(10-11)^2$	M1
	$r^2=50$	A1
	$"50"=(x\pm"(-1)")^2+(y\pm"10")^2$	M1
	$"50"=(x-"(-1)")^2+(y-"10")^2$	A1ft
	$x^2+y^2+2x-20y+51=0$	A1 **[5]** **(14 marks)**
	Alternative methods to part (d) (i)Use equation $x^2+y^2+ax+by+c=0$ and substitute three points, usually (0,3), (6,11) and another point on the circle maybe (-2,17) or (-8,9) - **not** point Z Solves simultaneous equations a = 2, b = -20 and c = 51	M1 dM1 A1,A1,A1
	(ii) Uses centre to write a = and b = (doubles x coordinate and y coordinate respectively, $\pm"2"$ *and* $\pm"20"$)	M1
	Obtains a= 2 and b = -20 (or just writes these values down so these answers imply M1A1)	A1
	Completes method to find c, (could substitute one of the points on the circle) or could find r	dM1
	Accurate work e.g. $r^2=50$ or e.g. $x^2+y^2+2x-20y=(-8)^2+9^2+2\times-8-20\times9=$	A1
	$c=51$	A1

Question number	Scheme	Marks
7 (a)	Obtain $(x \pm 5)^2$ **and** $(y \pm 3)^2$ Centre is $(-5, 3)$.	M1 A1 [2]
(b)	See $(x \pm 5)^2 + (y \pm 3)^2 = 16 (= r^2)$ or $(r^2 =)"25"+"9"-18$ $r = 4$	M1 A1 [2]
(c)	Use $x = -3$ in either form of equation of circle to obtain simplified quadratic in y e.g $x = -3 \Rightarrow (-3+5)^2 + (y-3)^2 = 16 \Rightarrow (y-3)^2 = 12$ *or* $(-3)^2 + y^2 + 10\times(-3) - 6y + 18 = 0 \Rightarrow y^2 - 6y - 3 = 0$ solve resulting quadratic to give $y =$ $y = 3 \pm 2\sqrt{3}$	M1 M1 A1, A1 [4]
		8 marks
Alternatives (a) OR	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$ Centre is $(-g, -f)$ **,** **and so centre is** $(-5, 3)$. *Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "-5" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "3" (Centre – chord theorem) . (-5, 3) is M1A1	M1 A1 M1 A1 (2)
(b)	*Method 2*: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)"25"+"9"-18$ $r = 4$	M1 A1 (2)
(c)	*Method 2*: Divide triangle PTQ and use Pythagoras with $r^2 - (-3 - "-5")^2 = h^2$, then evaluate $"3 \pm h"$ - then get $3 \pm 2\sqrt{3}$	M1 M1 A1 A1 (4)
	Notes	

Question Number	Scheme	Marks
12(a)	Writes C as $(x-a)^2+(y-0)^2=a^2$	M1A1 **(2)**
(b)	Subs $(4,-3)$ $\Rightarrow(4-a)^2+(-3-0)^2=a^2$	M1
	$\Rightarrow 16-8a+a^2+9=a^2$	
	$\Rightarrow 25=8a$	
	$\Rightarrow a=\dfrac{25}{8}$	dM1A1 **(3)**
		(5 marks)

Mark parts (a) and (b) together. Award marks in (a) from (b) and vice versa, but see note

(a)

M1 Attempts to find the equation of C centre $(a,0)$ radius a. Accept $(x\pm a)^2+y^2=a^2$ oe

If the alternative form of the circle is used accept $x^2+y^2\pm 2ax=a^2-a^2$

Allow for the M1 $(x\pm a)^2+(y\pm 0)^2=r^2$

A1 Writes C as $(x-a)^2+(y-0)^2=a^2$ or equivalent $x^2+y^2-2ax=0$.

(b)

M1 Subs $x=4$ and $y=-3$ into their circle equation for C which must be of the form $(x\pm a)^2+(y\pm 0)^2=a^2$

dM1 Proceeds to a linear equation in 'a' and reaches a=.... Condone numerical slips

A1 $a=\dfrac{25}{8}$ Accept exact alternatives

Note: There are some candidates who write the equation of the circle as $(x-a)^2+(y-0)^2=r^2$ in part (a)

This is M1 A0

However in part (b) they substitute $(4,-3)$ and write down $(4-a)^2+(-3)^2=a^2$

We will allow them to score all 3 marks in part (b).

Had they written $(x-a)^2+y^2=a^2$ in (b) we would allow them to score all 5 marks

Question	Scheme		Marks
13 (a)	See $(x\pm1)^2+(y\pm3)^2=r^2$	Or see $x^2+y^2\pm2x\pm6y+c=0$	M1
	Attempt $\sqrt{(8-1)^2+(-2-(-3))^2}$ **or** $(8-1)^2+(-2-(-3))^2$	Substitute $(8,-2)$ into equation	M1
	$(x-1)^2+(y+3)^2,=50$	$x^2+y^2-2x+6y-40=0$	A1, A1 **[4]**
(b)	Gradient of $AP=\frac{1}{7}$		B1
	So gradient of tangent is -7		M1
	Equation of tangent is $(y+2)=-7(x-8)$		dM1
	$y=-7x+54$ or $m=-7$, $c=54$		A1 **[4]**
	Way 1	Way 2	
(c)	$y=x+6$ meets circle when $(x-1)^2+(x+9)^2=50$ or when $(y-7)^2+(y+3)^2=50$	As tangent has gradient 1 AQ has gradient -1 and $\frac{y-(-3)}{x-1}=-1$	M1
	i.e. $2x^2+16x+32=0$ **or** when $2y^2-8y+8=0$	$y+x=-2$	A1
	Solve to give x or $y=$	Solve $y+x=-2$ with $y=x+6$ or alternatively solve $y+x=-2$ with the equation of the circle to give x or $y=$	M1
	Substitute to give $y=$ (**or** $x=$)		dM1
	$(-4, 2)$ only		A1 **[5]**
			13 marks

Notes

(a)

M1 : Scored for centre at $(1,-3)\Rightarrow(x\pm1)^2+(y\pm3)^2=\ldots$ or $x^2+y^2\pm2x\pm6y+\ldots=0$

M1: Scored for an attempt at finding the radius or the radius ^{2} (see scheme).
It need not be in the equation It can be implied by $\sqrt{50}$ *or* $5\sqrt{2}$ or 50
If the form $x^2+y^2\pm2x\pm6y+c=0$ is used it is for substituting $(8,-2)$ into the equation

A1: LHS or RHS correct $(x-1)^2+(y+3)^2,=\ldots$ or $(x\pm a)^2+(y\pm b)^2,=50$ $x^2+y^2-2x+6y\ldots=0$

A1: Correct equation. Accept $(x-1)^2+(y+3)^2=50$ or $x^2+y^2-2x+6y-40=0$ or $x^2+y^2-2x+6y=40$

(b)

B1 : Obtain 1/7 . Implied by use of – 7 in their tangent

M1: Uses negative reciprocal

dM1: Linear equation through point (8, -2) with their negative reciprocal gradient

A1: cao

(c)

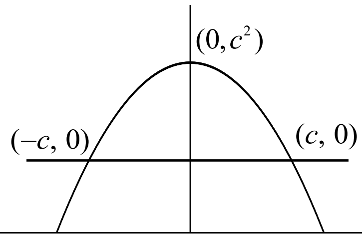
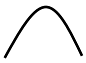
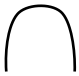
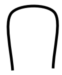
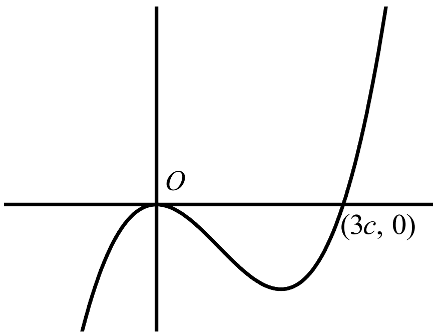
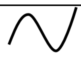
M1: Eliminates x or y from two relevant equations, that is whose intersection is Q.

A1: Correct quadratic in x or in y

M1: Solves (with usual rules) to give first variable. The first M must have been scored

dM1: Substitute in either (relevant) equation to give second coordinate, dependent upon both previous M's

A1: Correct answer accept $x=-4$, $y=2$. Withhold this if two answers given

Question Number	Scheme	Marks
13(a)(i)		
	 Or  shape anywhere but not  The maximum must be smooth and not form a point and the branches must not clearly turn back in on themselves. **or** A continuous graph passing through or touching at the points (-c, 0), (c, 0) and (0, c^2). They can appear on their sketch or within the body of the script but there must be a sketch. Allow these marked as $-c$, c and c^2 in the correct places. Allow (0, -c), (0, c) and (c^2, 0) as long as they are marked in the correct places. If there is any ambiguity, the sketch takes precedence.	B1
	A fully correct diagram with the curve in the correct position and the intercepts and shape as described above. The maximum must be on the y-axis and the branches must extend below the x-axis.	B1
(a)(ii)	**There must be a sketch to score any marks in (a)**	
	 —  Shape. A positive cubic with only one maximum and one minimum. The curve must be smooth at the maximum and at the minimum (not pointed).	B1
	A smooth curve that touches or meets the x-axis at the origin and $(3c, 0)$ in the correct place and no other intersections. The origin does not need to be marked but the $(3c, 0)$ does. Allow $3c$ or $(0, 3c)$ to be marked in the correct place. May appear on their sketch or within the body of the script. If there is any ambiguity, the sketch takes precedence.	B1
	Maximum at the origin (allow the maximum to form a point or cusp)	B1
	There must be a sketch to score any marks in (a)	**(5)**
(b)	Intersect when $x^2(x-3c) = c^2 - x^2 \Rightarrow x^3 - 3cx^2 = c^2 - x^2$ Sets equations equal to each other and attempts to multiply out the bracket or vice versa	M1
	$x^3 + x^2 - 3cx^2 - c^2 = 0$ $\Rightarrow x^3 + (1-3c)x^2 - c^2 = 0$* Collects to one side (may be implied), factorises the x^2 terms and obtains printed answer with no errors. **There must be an intermediate line of working.** **Allow** $x^3 + x^2(1-3c) - c^2 = 0$ or $0 = x^3 + (1-3c)x^2 - c^2$ or $0 = x^3 + x^2(1-3c) - c^2$	A1*
		(2)

(c)	$8+4(1-3c)-c^2=0$	Substitutes $x=2$ to give a **correct** un-simplified form of the equation.	M1
	$c^2+12c-12=0$	Correct 3 term quadratic. Allow any equivalent form with the terms collected (may be implied)	A1
	$(c+6)^2-36-12=0 \Rightarrow c=...$ or $c=\dfrac{-12\pm\sqrt{12^2-4\times1\times(-12)}}{2}$	Solves their 3TQ by using the formula or completing the square **only**. This may be implied by a correct **exact** answer for their 3TQ. (May need to check)	M1
	$4\sqrt{3}-6$	$c=4\sqrt{3}-6$ or $c=-6+4\sqrt{3}$ **only**	A1
			(4)
			(11 marks)

Question Number	Scheme	Marks
14 (a)	$\left(\frac{1+7}{2},\frac{4+8}{2}\right)=(4,6)$	M1A1
		(2)
(b)	$\frac{\sqrt{(7-1)^2+(8-4)^2}}{2}$ Or $\sqrt{('4'-1)^2+('6'-4)^2}$ Or $\sqrt{(7-'4')^2+(8-'6')^2}$	M1
	(Radius of circle) = $\sqrt{13}$	A1
		(2)
(c)	Equation of C_2 is $x^2+y^2=r^2$	M1
	Attempts either value of r as $\left(\sqrt{'4'^2+'6'^2}\pm \text{their } r\right)$	M1
	When $r=\sqrt{52}-\sqrt{13}=\sqrt{13} \Rightarrow x^2+y^2=13$	A1
	When $r=\sqrt{52}+\sqrt{13}=3\sqrt{13} \Rightarrow x^2+y^2=117$	A1
		(4) **(8marks)**

(a)

M1 For an attempt at $\left(\frac{1+7}{2},\frac{4+8}{2}\right)$ May be implied by either correct coordinate

A1 $(4,6)$. No working is required, Correct answer scores both marks. Condone lack of brackets

(b)

M1 Scored for using Pythagoras' theorem to find the distance between their centre and a point. Look for an attempt at $\sqrt{('4'-1)^2+('6'-4)^2}$ or similar. If the original coordinates are used then there must be some attempt to halve.

A1 $=\sqrt{13}$ Correct answer scores both marks

(c)

M1 For stating the equation of C_2 is $x^2+y^2=r^2$ or $(x-0)^2+(y-0)^2=r^2$ for any 'r' including an algebraic 'r' Accept $x^2+y^2=k$ If a value of k is given then k must be positive

M1 Attempts either value of r Look for $\left(\sqrt{'4'^2+'6'^2}\pm \text{their } r\right)$ Accept $r=\frac{\sqrt{4^2+6^2}}{2}$

A1 Either of $x^2+y^2=13$ or $x^2+y^2=117$

Allow for this mark variations like $(x-0)^2+(y-0)^2=\sqrt{13}^2$

A1 Both of $x^2+y^2=13$ and $x^2+y^2=117$. Equations must be simplified as seen here

Any one correct equation will imply the first two M's.

……

Alt method to find equations using the intersections:

M1: As above

M1: Solves 'their' $y='\frac{3}{2}'x$ with their $(x-'4')^2+(y-'6')^2='13' \Rightarrow$ Intersections $(2,3)$ and $(6,9)$

So this time the method is scored for either $\sqrt{'2'^2+'3'^2}$ or $\sqrt{'6'^2+'9'^2}$

A1 A1 as before

Question Number	Scheme	Marks
13 (a)(i)	$(3,-4)$	B1
(a)(ii)	$\sqrt{30}$	B1
		[2]
(b)	Attempts $(6-3)^2+(k+4)^2, < 30$	M1,M1
	$k^2+8k-5<0$	A1*
		[3]
(c)	Solves quadratic by formula or completion of square to give $k=$	M1
	$k=-4\pm\sqrt{21}$	A1
	Chooses region between two values and deduces $-4-\sqrt{21}<k<-4+\sqrt{21}$	M1
		A1**cao**
		[4]
		(9 marks

(a)(i)(ii)

B1 $(3,-4)$ Accept as $x=\ ,y=$ or even without the brackets

B1 $\sqrt{30}$ Do not accept decimals here but remember to isw

(b) This is scored M1 A1 A1 on e -pen. We are marking it M1 M1 A1

M1 Attempts to find the length or length^2 from P $(6,k)$, to the centre of C $(3,-4)$ following through on their C. Look for, using a correct C, either $(6-'3')^2+(k+'4')^2$ or $\sqrt{(6-'3')^2+(k+'4')^2}$

Another way is to substitute $(6,k)$ into $(x-3)^2+(y+4)^2=30$ but it is very difficult to score either of the other two marks using this method.

M1 Forms an inequality by using the length from P to the centre of C < the radius of C
$(6-3)^2+(k+4)^2<30$. In almost all cases I would expect to see < 30 before < 0

Using the alternative method, they would also need the line $(6-3)^2+(k+4)^2<30$. (As if the point lies on another circle, the radius/distance would need to be smaller than 30)

A1* $k^2+8k-5<0$
This is a given answer and you must check that all aspects are correct. **In most cases you should expect to see an intermediate line (with < 30) before the final answer appear with < 0.**

(c)

M1 Solves the equation $k^2+8k-5=0$ by formula or completing the square.
Factorisation to integer roots is not a suitable method in this case and scores M0.
The answers could just appear from a graphical calculator. Accept decimals for the M's only

A1 Accept $k=-4\pm\sqrt{21}$ or exact equivalent $k=\dfrac{-8\pm\sqrt{84}}{2}$

Do not accept decimal equivalents $k=-8.58,(+)0.58$ 2dp for this mark

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation.

A1 cao $-4-\sqrt{21}<k<-4+\sqrt{21}$ Accept equivalents such as $(-4-\sqrt{21},-4+\sqrt{21})$,
$k>-4-\sqrt{21}$ and $k<-4+\sqrt{21}$, even $k>-4-\sqrt{21}$, $k<-4+\sqrt{21}$

Accept for 3 out of 4 $[-4-\sqrt{21},-4+\sqrt{21}]$, $k>-4-\sqrt{21}$ or $k<-4+\sqrt{21}$, $-4-\sqrt{21}\leqslant k\leqslant -4+\sqrt{21}$

Do not accept $-4-\sqrt{21}<x<-4+\sqrt{21}$ for this final mark

Question Number	Scheme		Marks
3.	$P(7,8)$ and $Q(10,13)$		
(a)	$\{PQ=\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) **Way 1**	$(x-7)^2+(y-8)^2=34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x\pm 7)^2+(y\pm 8)^2=k$, where k is a positive value.	M1
		$(x-7)^2+(y-8)^2=34$	A1 oe
			[2]
(b) **Way 2**	$x^2+y^2-14x-16y+79=0$	$x^2+y^2\pm 14x\pm 16y+c=0$, where c is any value <113.	M1
		$x^2+y^2-14x-16y+79=0$	A1 oe
			[2]
(c) **Way 1**	$\{\text{Gradient of radius}\}=\frac{13-8}{10-7}$ or $\frac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $=-\frac{1}{m}\left(=-\frac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $=-\frac{1}{\text{gradient of radius}}$	M1
	$y-13=-\frac{3}{5}(x-10)$	$y-13=(\text{their changed gradient})(x-10)$	M1
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
(c) **Way 2**	$2(x-7)+2(y-8)\frac{dy}{dx}=0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7)+2(13-8)\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{3}{5}$	Substituting **both** $x=10$ and $y=13$ into a valid differentiation to find a value for $\frac{dy}{dx}$	M1
	$y-13=-\frac{3}{5}(x-10)$	$y-13=(\text{their gradient})(x-10)$	M1
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
(c) **Way 3**	$10x+13y-7(x+10)-8(y+13)+79=0$	$10x+13y-7(x+10)-8(y+13)+79=0$	B1
		$10x+13y-7(x+10)-8(y+13)+c=0$ where c is any value <113	M2
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
			8

Question Number	Scheme		Marks
	Mark (a) and (b) together		
9. (a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{=14\}$	Uses the addition form of Pythagoras on $6\sqrt{5}$ and 4. Condone missing brackets on $\left(6\sqrt{5}\right)^2$ **(Working or 14 may be seen on the diagram)**	M1
	$y_Q = \sqrt{14^2 - 11^2}$	$y_Q = \sqrt{(\text{their } OQ)^2 - 11^2}$ **Must include √ and is dependent on the first M1 and requires OQ > 11**	**d**M1
	$= \sqrt{75}$ or $5\sqrt{3}$	$\sqrt{75}$ or $5\sqrt{3}$	A1**cso**
			[3]
(b)	$(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1: $(x \pm 11)^2 + (y \pm \text{their } k)^2 = 4^2$ **Equation must be of this form and must use *x* and *y* not other letters. *k* could be their last answer to part (a). Allow their *k* ≠ 0 or just the letter *k*.** A1: $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ or $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 4^2$ **NB** $5\sqrt{3}$ **must come from correct work in (a) and allow awrt 8.66**	M1A1
	Allow in expanded form for the final A1 e.g. $x^2 - 22x + 121 + y^2 - 10\sqrt{3}y + 75 = 16$		
			[2]
			Total 5
	Watch out for:		
	(a) $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2} = \sqrt{46}$ M1 $y_Q = \sqrt{46 - 11^2}$ M0 (OQ < 11) $y_Q = \sqrt{75}$ A0 (b) $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ M1A0		

5.			
(a)			
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r=...$		M1
	Completes the square for both x and y in an attempt to find r. $(x\pm"10")^2 \pm a$ and $(y\pm"12")^2 \pm b$ and $+195=0, (a,b\neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r=\sqrt{(\pm"10")^2+(\pm"12")^2-c}$		M1
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r = 7$		A1
			(5)
(b)	$MN=\sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP=\sqrt{("25"^2-"7"^2)}$	$NP=\sqrt{(MN^2-r^2)}$	M1
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP)=\frac{7}{"25"} \Rightarrow NP="25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP = 24$		A1
			(2)
			[9]

Question number	Scheme	Marks
3 (a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	Obtain $(x - 10)^2$ **and** $(y - 8)^2$	A1
	Centre is $(10, 8)$. N.B. This may be indicated on **diagram only** as (10, 8)	A1 (3)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 =)$"100"+"64"-139	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	**A1** **(2)**
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	**M1**
	e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y=$ *or* $13^2 + y^2 - 20\times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$	
	$y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**A1, A1** (3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	**M1**
	Perimeter *PTQ* = 2*r* + their **arc** *PQ* (Finding perimeter of triangle is M0 here)	**M1**
	= 19.275 or 19.28 or 19.3	**A1** (3)
		11 marks
Alternatives (a)	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$ Centre is $(-g, -f)$, **and so centre is** $(10, 8)$.	M1 A1, A1
OR	*Method 3:* Use any value of y to give two points (*L* and *M*) on circle. *x* co-ordinate of mid point of *LM* is "10" **and** Use any value of *x* to give two points (*P* and *Q*) on circle. *y* co-ordinate of mid point of *PQ* is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	*Method 2*: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)$"100"+"64"-139 $r = 5$ *	M1 A1
OR	*Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$ $r = 5$ *	**M1** **A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate "$8 \pm h$" - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**
Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be implied by $(\pm 10, \pm 8)$ **. Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 =$ "100"+"64"-139 (not 139 – "100" –" 64") or for using equation of circle in $(x \pm 10)^2 + (y \pm 8)^2 = k^2$ form to identify $r=$ **3rd A1** ***r*** **= 5 (NB This is a given answer so should follow** $k^2 = 25$ or $r^2 = 100 + 64 - 139$) **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

Question	Scheme	Marks	AOs
8 (a) Way 1	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x=20,\ H=12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x=40,\ H=0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x=0,\ H=0 \Rightarrow 0 = 0+0+c \Rightarrow c=0$ **and either** $x=40,\ H=0 \Rightarrow 0 = 1600a + 40b$ **or** $x=20,\ H=12 \Rightarrow 12 = 400a + 20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b=-40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H=3 \Rightarrow\}\ 3 = \frac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H=3 \Rightarrow\}\ 3 = 12 - \frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20 + \sqrt{300} \Rightarrow\right\}$ greatest distance $=$ awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) Way 1	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 2	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \frac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 3	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} (Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss $= 1.12$} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit $= 0.04$} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)