14. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

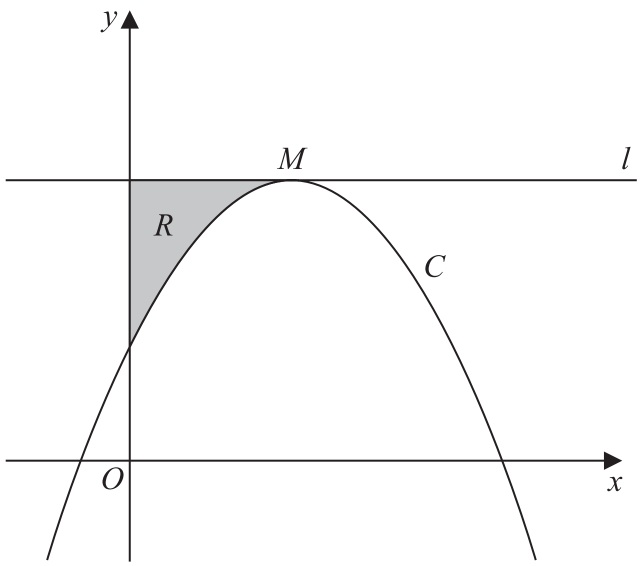

Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

9. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = 20$$

(4)

P 6 6 5 8 5 A 0 1 8 4 4

3. Find

$$\int \frac{3x^4 - 4}{2x^3} \, \mathrm{d}x$$

writing your answer in simplest form.

(4)

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

15.

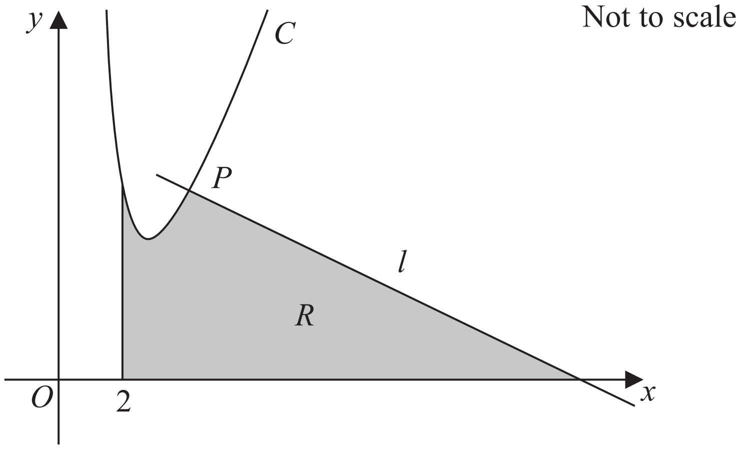

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) dx = 8$$

(3)

13.

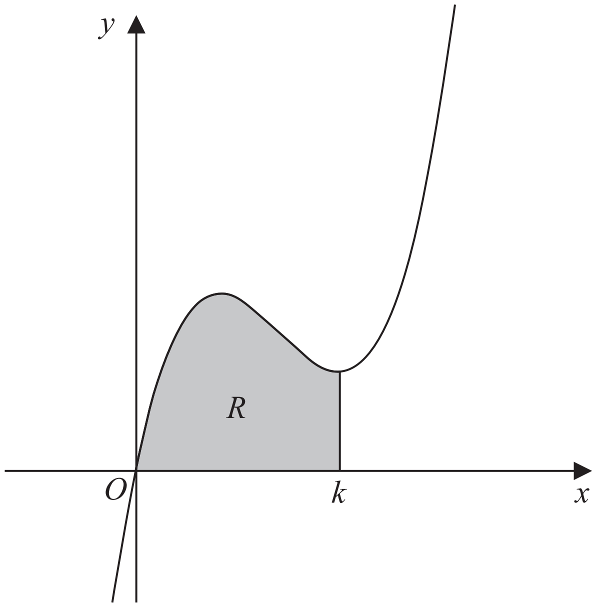

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

7. Given that k is a positive constant and $\displaystyle\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

(4)

10. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that g(x) is divisible by (x – 5).

(2)

(b) Hence, showing all your working, write g(x) as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

(Total for Question 5 is 5 marks)

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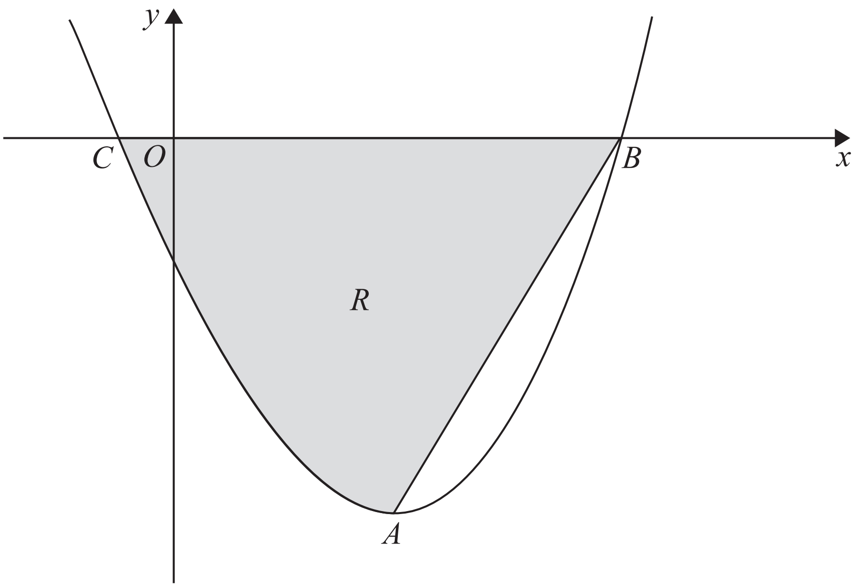

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

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7.

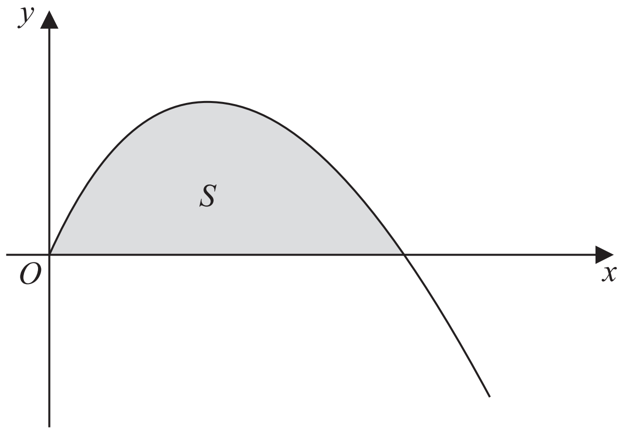

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)

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3. (a) Express $\dfrac{x^3 + 4}{2x^2}$ in the form $Ax^p + Bx^q$, where A, B, p and q are constants.

(3)

(b) Hence find

$$\int \frac{x^3 + 4}{2x^2}\, \mathrm{d}x$$

simplifying your answer.

(3)

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15.

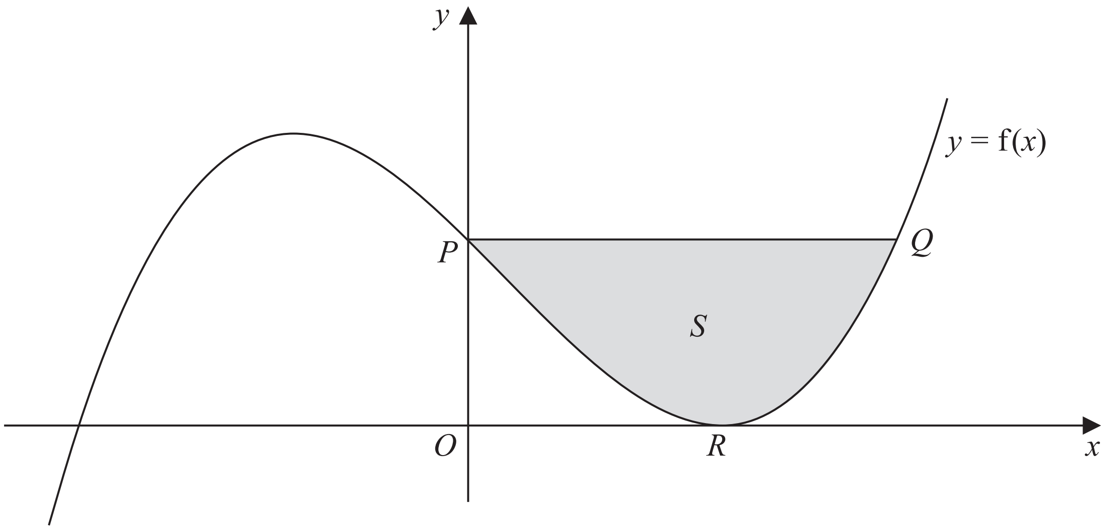

Figure 5

Figure 5 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{(x-3)^2(x+4)}{2}, \qquad x \in \mathbb{R}$$

The graph cuts the y-axis at the point P and meets the positive x-axis at the point R, as shown in Figure 5.

(a) (i) State the y coordinate of P.

(ii) State the x coordinate of R.

(2)

The line segment PQ is parallel to the x-axis. Point Q lies on $y = \mathrm{f}(x)$, $x > 0$

(b) Use algebra to show that the x coordinate of Q satisfies the equation

$$x^2 - 2x - 15 = 0$$

(3)

(c) Use part (b) to find the coordinates of Q.

(3)

The region S, shown shaded in Figure 5, is bounded by the curve $y = \mathrm{f}(x)$ and the line segment PQ.

(d) Use calculus to find the exact area of S.

(6)

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8. (a) Find $\int (3x^2 + 4x - 15)\,dx$, simplifying each term.

(3)

Given that b is a constant and

$$\int_b^4 (3x^2 + 4x - 15)\,dx = 36$$

(b) show that $b^3 + 2b^2 - 15b = 0$

(2)

(c) Hence find the possible values of b.

(3)

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12.

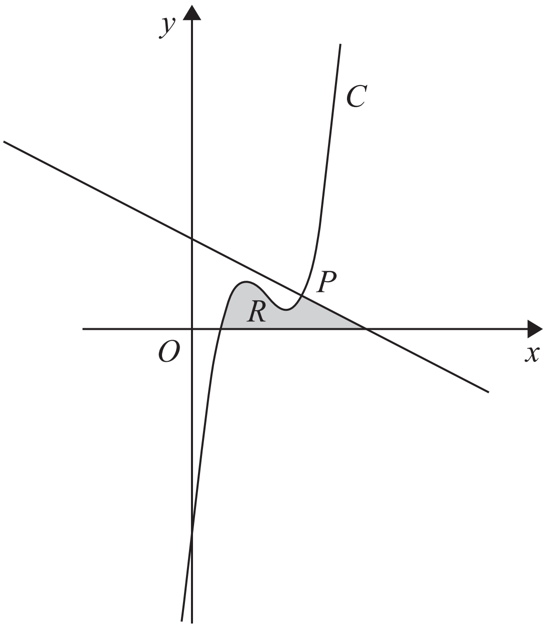

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = x^3 - 9x^2 + 26x - 18$$

The point P (4, 6) lies on C.

(a) Use calculus to show that the normal to C at the point P has equation

$$2y + x = 16$$

(5)

The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the normal to C at P.

(b) Show that C cuts the x-axis at $(1, 0)$

(1)

(c) Showing all your working, use calculus to find the exact area of R.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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7. (i) Find

$$\int \frac{2 + 4x^3}{x^2}\,dx$$

giving each term in its simplest form.

(4)

(ii) Given that k is a constant and

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k\right) dx = 30$$

find the exact value of k.

(5)

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12.

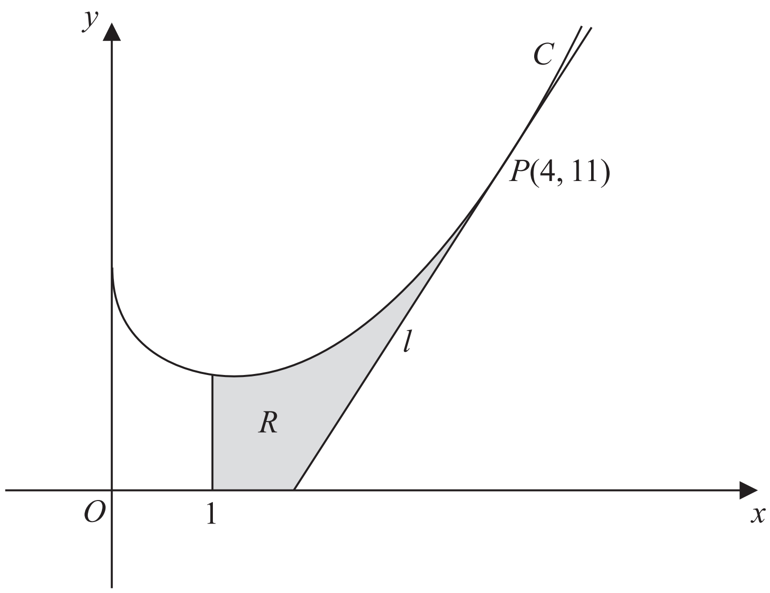

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7, \quad x > 0$$

The point P lies on C and has coordinates (4, 11).

Line l is the tangent to C at the point P.

(a) Use calculus to show that l has equation $y = 5x - 9$

(5)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line $x = 1$, the x-axis and the line l.

(b) Find, by using calculus, the area of R, giving your answer to 2 decimal places.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

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1.

$$f(x) = 3x^2 + x - \frac{4}{\sqrt{x}} + 6x^{-3}, \quad x > 0$$

Find $\int f(x)\,dx$, simplifying each term.

(5)

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14.

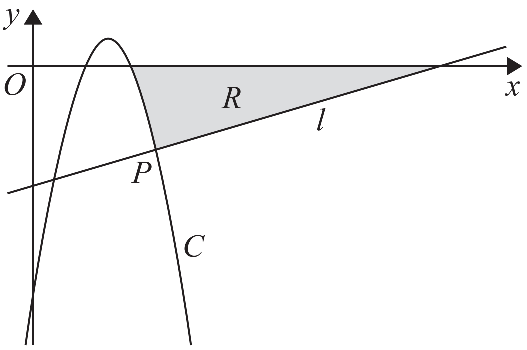

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = -x^2 + 6x - 8$. The normal to C at the point $P(5, -3)$ is the line l, which is also shown in Figure 3.

(a) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The finite region R, shown shaded in Figure 3, is bounded below by the line l and the curve C, and is bounded above by the x-axis.

(b) Find the exact value of the area of R.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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6. (a) Show that $\dfrac{x^2 - 4}{2\sqrt{x}}$ can be written in the form $Ax^p + Bx^q$, where A, B, p and q are constants to be determined.

(3)

(b) Hence find

$$\int \frac{x^2 - 4}{2\sqrt{x}}\,\mathrm{d}x, \quad x > 0$$

giving your answer in its simplest form.

(4)