6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

9. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

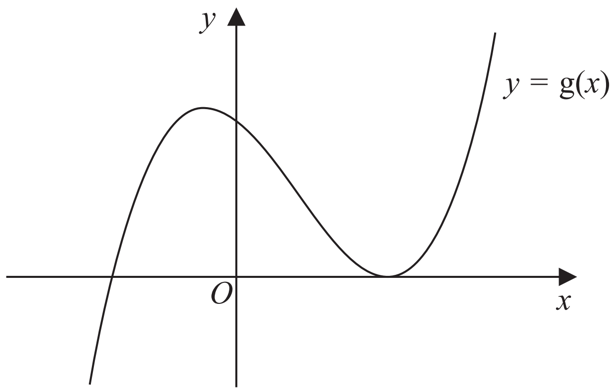

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

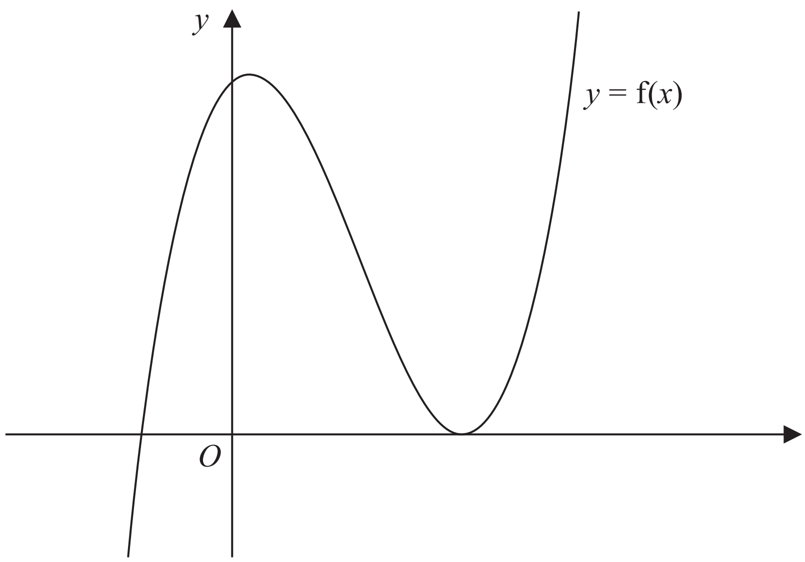

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 4 is 6 marks)

13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

Leave blank

6. $$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

Leave blank

2. The point $P(2, 3)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of P under the transformation represented by the curve with equation

(a) $y = f(x + 2)$

(1)

(b) $y = -f(x)$

(1)

(c) $2y = f(x)$

(1)

(d) $y = f(x) - 4$

(1)

Leave blank

5. $$f(x) = -4x^3 + 16x^2 - 13x + 3$$

(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(x - 1)$. **(2)**

(b) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. **(2)**

(c) Hence fully factorise $f(x)$. **(4)**

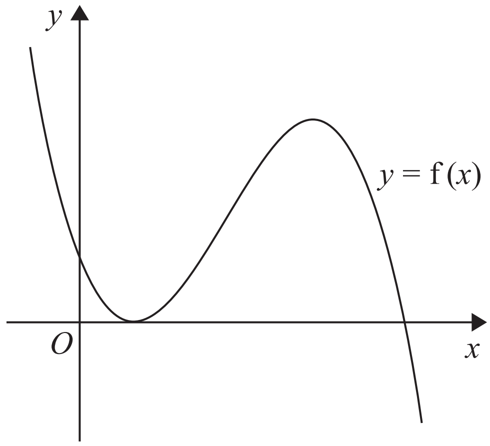

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

(d) Use your answer to part (c) and the sketch to deduce the set of values of x for which $f(x) \leqslant 0$ **(2)**

Leave blank

7.

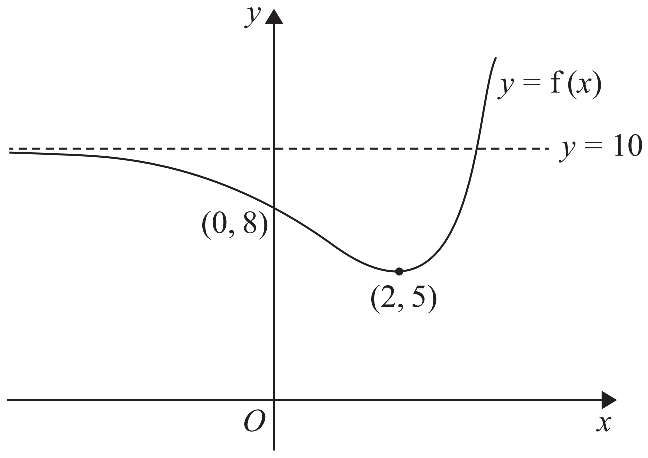

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve crosses the y-axis at the point (0, 8).

The line with equation $y = 10$ is the only asymptote to the curve.

The curve has a single turning point, a minimum point at (2, 5), as shown in Figure 3.

(a) State the coordinates of the minimum point of the curve with equation $y = \mathrm{f}\left(\frac{1}{4}x\right)$

(1)

(b) State the equation of the asymptote to the curve with equation $y = \mathrm{f}(x) - 3$

(1)

The curve with equation $y = \mathrm{f}(x)$ meets the line with equation $y = k$, where k is a constant, at two distinct points.

(c) State the set of possible values for k.

(2)

(d) Sketch the curve with equation $y = -\mathrm{f}(x)$. On your sketch, show clearly the coordinates of the turning point, the coordinates of the intersection with the y-axis and the equation of the asymptote.

(3)

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9.

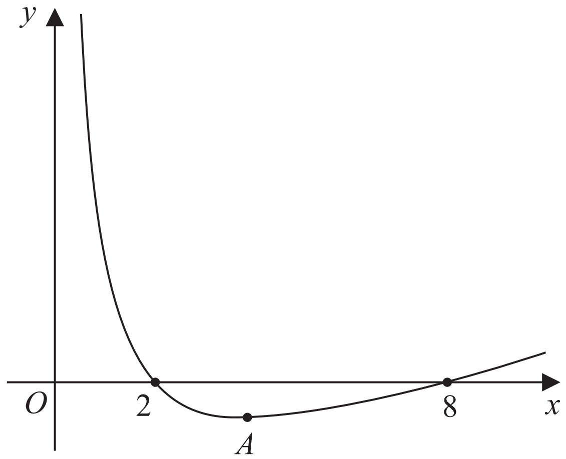

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{8}{x} + \frac{1}{2}x - 5, \qquad 0 < x \leqslant 12$$

The curve crosses the x-axis at (2, 0) and (8, 0) and has a minimum point at A.

(a) Use calculus to find the coordinates of point A.

(5)

(b) State

(i) the roots of the equation $2\mathrm{f}(x) = 0$

(ii) the coordinates of the turning point on the curve $y = \mathrm{f}(x) + 2$

(iii) the roots of the equation $\mathrm{f}(4x) = 0$

(3)

P 4 8 3 2 4 A 0 2 6 5 2

Leave blank

13. (a) On separate axes sketch the graphs of

(i) $y = c^2 - x^2$

(ii) $y = x^2(x - 3c)$

where c is a positive constant.

Show clearly the coordinates of the points where each graph crosses or meets the x-axis and the y-axis.

(5)

(b) Prove that the x coordinate of any point of intersection of

$$y = c^2 - x^2 \quad \text{and} \quad y = x^2(x - 3c)$$

where c is a positive constant, is given by a solution of the equation

$$x^3 + (1 - 3c)x^2 - c^2 = 0$$

(2)

Given that the graphs meet when $x = 2$

(c) find the exact value of c, writing your answer as a fully simplified surd.

(4)

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14.

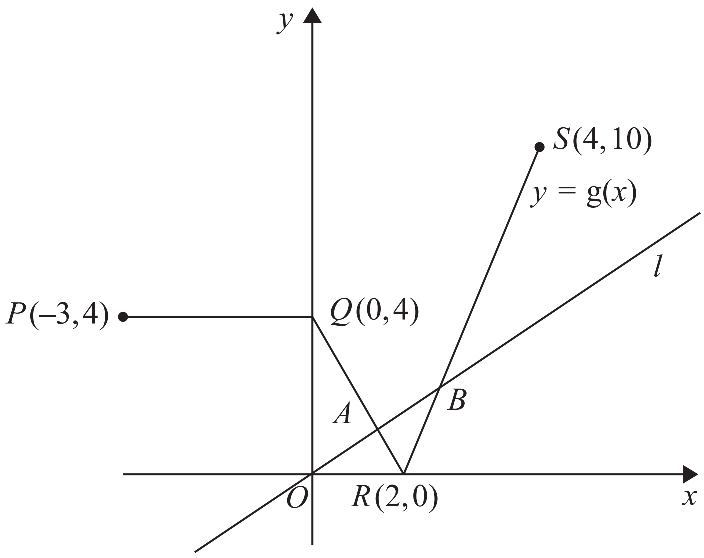

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, $-3 \leqslant x \leqslant 4$ and part of the line l with equation $y = \frac{1}{2}x$

The graph of $y = \mathrm{g}(x)$ consists of three line segments, from $P(-3, 4)$ to $Q(0, 4)$, from $Q(0, 4)$ to $R(2, 0)$ and from $R(2, 0)$ to $S(4, 10)$.

The line l intersects $y = \mathrm{g}(x)$ at the points A and B as shown in Figure 4.

(a) Use algebra to find the x coordinate of the point A and the x coordinate of the point B.

Show each step of your working and give your answers as exact fractions.

(6)

(b) Sketch the graph with equation

$$y = \frac{3}{2}\mathrm{g}(x), \quad -3 \leqslant x \leqslant 4$$

On your sketch show the coordinates of the points to which P, Q, R and S are transformed.

(2)

Leave blank

6. $$f(x) = x^3 + x^2 - 12x - 18$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$.

(2)

(c) Hence find exact values for all the solutions of the equation $f(x) = 0$

(3)