Question	Scheme	Marks	AOs
9(a)(i)	Weight = mass × g $\Rightarrow m = \frac{30\,000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	M1	3.3
(ii)	$\frac{dx}{dt} = 40\cos t + 20\sin t,\ \frac{d^2x}{dt^2} = -40\sin t + 20\cos t$	M1	1.1b
	$3(-40\sin t + 20\cos t) + 4(40\cos t + 20\sin t)$ $+ 40\sin t - 20\cos t = ...$	M1	1.1b
	$= 200\cos t$ so PI is $x = 40\sin t - 20\cos$	A1*	2.1
	or		
	Let $x = a\cos t + b\sin t$ $\frac{dx}{dt} = -a\sin t + b\cos t,\ \frac{d^2x}{dt^2} = -a\cos t - b\sin t$	M1	1.1b
	$4b - 2a = 200,\ -2b - 4a = 0 \Rightarrow a = ..., b = ...$	M1	2.1
	$x = 40\sin t - 20\cos t$	A1*	1.1b
(iii)	$3\lambda^2 + 4\lambda + 1 = 0 \Rightarrow \lambda = -1, -\frac{1}{3}$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t}$	A1	1.1b
	$x = PI + CF$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
		(8)	
(b)	$t = 0, x = 0 \Rightarrow A + B = 20$	M1	3.4
	$x = 0, \frac{dx}{dt} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40\cos t + 20\sin t = 0$ $\Rightarrow A + \frac{1}{3}B = 40$	M1	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40\sin t - 20\cos t$	A1	1.1b
	$t = 9 \Rightarrow x = 33\text{m}$	A1	3.4
		(4)	
			(12 marks)

Question	Scheme	Marks	AOs
7(a)	$r=10\frac{\mathrm{d}f}{\mathrm{d}t}-2f \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t}=10\frac{\mathrm{d}^2f}{\mathrm{d}t^2}-2\frac{\mathrm{d}f}{\mathrm{d}t}$	M1	2.1
	$10\frac{\mathrm{d}^2f}{\mathrm{d}t^2}-2\frac{\mathrm{d}f}{\mathrm{d}t}=-0.2f+0.4\left(10\frac{\mathrm{d}f}{\mathrm{d}t}-2f\right)$	M1	2.1
	$\frac{\mathrm{d}^2f}{\mathrm{d}t^2}-0.6\frac{\mathrm{d}f}{\mathrm{d}t}+0.1f=0$*	A1*	1.1b
		(3)	
(b)	$m^2-0.6m+0.1=0\Rightarrow m=\frac{0.6\pm\sqrt{0.6^2-4\times0.1}}{2}$	M1	3.4
	$m=0.3\pm0.1\mathrm{i}$	A1	1.1b
	$f=\mathrm{e}^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)$	M1	3.4
	$f=\mathrm{e}^{0.3t}\left(A\cos0.1t+B\sin0.1t\right)$	A1	1.1b
		(4)	
(c)	$\frac{\mathrm{d}f}{\mathrm{d}t}=0.3\mathrm{e}^{0.3t}\left(A\cos0.1t+B\sin0.1t\right)+0.1\mathrm{e}^{0.3t}\left(B\cos0.1t-A\sin0.1t\right)$	M1	3.4
	$r=10\frac{\mathrm{d}f}{\mathrm{d}t}-2f$ $=\mathrm{e}^{0.3t}\left((3A+B)\cos0.1t+(3B-A)\sin0.1t\right)-2\mathrm{e}^{0.3t}\left(A\cos0.1t+B\sin0.1t\right)$	M1	3.4
	$r=\mathrm{e}^{0.3t}\left((A+B)\cos0.1t+(B-A)\sin0.1t\right)$	A1	1.1b
		(3)	
(d)(i)	$t=0, f=6\Rightarrow A=6$	M1	3.1b
	$t=0, r=20\Rightarrow B=14$	M1	3.3
	$r=\mathrm{e}^{0.3t}\left(20\cos0.1t+8\sin0.1t\right)=0$	M1	3.1b
	$\tan0.1t=-2.5$	A1	1.1b
	2019	A1	3.2a
(d)(ii)	3750 foxes	B1	3.4
(d)(iii)	e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible	B1	3.5a
		(7)	
			(17 marks)

Question	Scheme	Marks	AOs
8(a)	$\frac{d^2w}{dt^2}=\frac{5}{2}\left(\frac{dw}{dt}-\frac{ds}{dt}\right)$ or $\frac{ds}{dt}=\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}$ o.e.	B1	1.1b
	$\frac{ds}{dt}=\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}\Rightarrow\frac{dw}{dt}-\frac{2}{5}\frac{d^2w}{dt^2}=\frac{2}{5}w-90e^{-t}$	M1	2.1
	$2\frac{d^2w}{dt^2}-5\frac{dw}{dt}+2w=450e^{-t}$ *	A1*	1.1b
		(3)	
(b)	$2m^2-5m+2=0\Rightarrow m=...$	M1	3.4
	$m=2,\frac{1}{2}$	A1	1.1b
	$(w)=Ae^{\alpha t}+Be^{\beta t}$	M1	3.4
	$(w)=Ae^{0.5t}+Be^{2t}$	A1	1.1b
	PI: Try $w=ke^{-t}\Rightarrow\frac{dw}{dt}=-ke^{-t}\Rightarrow\frac{d^2w}{dt^2}=ke^{-t}$ $2ke^{-t}+5ke^{-t}+2ke^{-t}=450e^{-t}\Rightarrow k=...$	M1	3.4
	$w=$ 'their C.F.' + $50e^{-t}$ $\left(w=Ae^{0.5t}+Be^{2t}+50e^{-t}\right)$	A1ft	1.1b
		(6)	
(c)	$s=w-\frac{2}{5}\frac{dw}{dt}=Ae^{0.5t}+Be^{2t}+50e^{-t}-\frac{2}{5}\left(\frac{A}{2}e^{0.5t}+2Be^{2t}-50e^{-t}\right)$	M1	3.4
	$s=\frac{4A}{5}e^{0.5t}+\frac{B}{5}e^{2t}+70e^{-t}$	A1	1.1b
		(2)	
(d)	$65=A+B+50,\ 85=\frac{4A}{5}+\frac{B}{5}+70\Rightarrow A=...,B=...$ $\left(\text{NB } A=20\ \ B=-5\right)$	M1	3.3
	$w=0\Rightarrow 20e^{0.5t}-5e^{2t}+50e^{-t}=0$	dM1	1.1b
	$e^{3t}-4e^{1.5t}-10(=0)$ or a multiple	A1	3.1a
	$e^{1.5t}=\frac{4\pm\sqrt{4^2-4\times(1)(-10)}}{2}$	M1	1.1b
	$1.5t=\ln\left(\frac{4+\sqrt{56}}{2}\right)$	M1	2.3
	$T=\frac{2}{3}\ln\left(\frac{4+\sqrt{56}}{2}\right)=$ awrt 1.165	A1	3.2a
		(6)	

(e)	E.g. • Either population becomes negative which is not possible • When the white-clawed crayfish have died out, the model will not be valid	B1	3.5b
		(1)	
			(18 marks)

Notes

(a)
B1: Differentiates the first equation with respect to t correctly.
M1: Substitutes $\frac{\mathrm{d}s}{\mathrm{d}t}$ into their derivative.
A1*: Achieves the printed answer with no errors.

(b) **Note: All the mark except the final A1 are available if they use other variables.**
M1: Uses the model to form and solve the Auxiliary Equation.
A1: Correct roots of the AE.
M1: Uses the model to form the Complementary Function for their roots (they may be complex roots)
A1: Correct CF
M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI. Uses $w = ke^{-t}$ finds both $\frac{\mathrm{d}w}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2w}{\mathrm{d}t^2}$ substitutes into the differential equation and find the value of k.
A1ft: Dependent on all three of the previous method marks. Following through on their CF only to give $w =$ 'their CF' $+ 50e^{-t}$

(c)
M1: Substitutes into the first equation the answer for part (b) in place of w and the derivative of their (b) in place of $\frac{\mathrm{d}w}{\mathrm{d}t}$. If they rearrange to make S the subject first and make a slip but still substitutes for w and $\frac{\mathrm{d}w}{\mathrm{d}t}$ allow this mark.
A1: Correct simplified equation.

(d)
M1: Uses the initial conditions $t = 0$, $w = 65$ and $s = 85$ to form simulations equations and solves to find the values of their constants
dM1: Dependent on the previous method mark. Sets $w = 0$
A1: Processes the indices correctly to obtain a 3-term quadratic equation in terms of $e^{1.5t}$. It does not need to all be on one side and condone missing = 0.
M1: Solves their three-term quadratic (3TQ) to reach $e^{pt} = q$
M1: Correct use of logarithms to reach $pt = \ln q$ where $q > 0$ and rejects the other solution
A1: awrt 1.165

Question	Scheme	Marks	AOs
5(a)	$4m^2+4m+37=0 \Rightarrow m=-\frac{1}{2}\pm 3\text{i}$	M1	1.1b
	$h=\text{e}^{-0.5t}\left(A\cos 3t+B\sin 3t\right)$	A1	1.1b
		(2)	
(b)	$t=0,\ h=-20 \Rightarrow A=-20$	M1	3.4
	$\frac{\text{d}h}{\text{d}t}=-0.5\text{e}^{-0.5t}\left(A\cos 3t+B\sin 3t\right)+\text{e}^{-0.5t}\left(-3A\sin 3t+3B\cos 3t\right)$ $t=0,\ \frac{\text{d}h}{\text{d}t}=55 \Rightarrow B=...$ (NB B = 15)	M1	3.4
	$\left(h=\right)\text{e}^{-0.5t}\left(15\sin 3t-20\cos 3t\right)$	A1	1.1b
	$-0.5\text{e}^{-0.5t}\left(15\sin 3t-20\cos 3t\right)+\text{e}^{-0.5t}\left(60\sin 3t+45\cos 3t\right)=0$ or e.g. $-0.5\text{e}^{-0.5t}\left(15\sin 3t-20\cos 3t\right)+\frac{25\sqrt{37}}{2}\text{e}^{-0.5t}\sin\left(3t+\arctan\frac{22}{21}\right)=0$ $\Rightarrow t=...$	M1	3.1b
	$\tan 3t=-\frac{22}{21}$ or e.g. $3t+\tan^{-1}\frac{22}{21}=0$	A1 **M1 on ePEN**	2.1
	t = 0.778 s	A1	1.1b
	$h=\text{e}^{-0.5\times"0.778"}\left(15\sin\left(3\times"0.778"\right)-20\cos\left(3\times"0.778"\right)\right)$	**d**M1	1.1b
	= 16.7 cm	A1	3.2a
		(8)	
(c)	E.g. considers large values of t in the model for h **or** states that for large values of t, h becomes smaller or becomes zero	M1	3.4
	E.g. • The value of h is very small when t is large and this is likely to be correct (as the displacement of end of the board should get smaller and smaller) • This suggests the model is suitable • This is realistic • This is suitable as the board will tend towards its equilibrium position • When t is large the value of h is never zero so the model is not really appropriate for large values of t	A1 **B1 on ePEN**	3.2b
		(2)	
			(12 marks)

Notes

(a)

M1: Uses the model to form and solve the auxiliary equation $4m^2+4m+37=0$

See General Guidance for awarding this mark. This can be implied by correct values for m (from calculator)

A1: Correct general solution including "h ="

(b)

Question	Scheme	Marks	AOs
5(a)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-5\frac{\mathrm{d}x}{\mathrm{d}t}+10\frac{\mathrm{d}y}{\mathrm{d}t}$ oe e.g. $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{10}\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}\right)$	B1	1.1b
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-5\frac{\mathrm{d}x}{\mathrm{d}t}+10(-2x+3y-4)$ $=-5\frac{\mathrm{d}x}{\mathrm{d}t}-20x+\frac{30}{10}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x+30\right)-40$ Or $\frac{1}{10}\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}\right)=-2x+\frac{3}{10}\left(30+5x+\frac{\mathrm{d}x}{\mathrm{d}t}\right)-4$	M1	2.1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+5x=50*$	A1*	1.1b
		(3)	
(b)	$m^2+2m+5=0\Rightarrow m=...$	M1	3.4
	$m=-1\pm 2\mathrm{i}$	A1	1.1b
	$m=\alpha\pm\beta\mathrm{i}\Rightarrow x=\mathrm{e}^{\alpha t}(A\cos\beta t+B\sin\beta t)=...$	M1	3.4
	$x=\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$	A1	1.1b
	PI: Try $x=k\Rightarrow 5k=50\Rightarrow k=10$	M1	3.4
	$GS: x=\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)+10$	A1ft	1.1b
		(6)	
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{-t}(2B\cos 2t-2A\sin 2t)-\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$	B1ft	1.1b
	$(y=)\frac{1}{10}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x+30\right)=...$	M1	3.4
	$y=\frac{1}{10}\mathrm{e}^{-t}\left((4A+2B)\cos 2t+(4B-2A)\sin 2t\right)+8$	A1	1.1b
		(3)	
(d)	$t=0, x=2\Rightarrow 2=A+10\Rightarrow A=-8$	M1	3.1b
	$t=0, y=5\Rightarrow 5=\frac{1}{10}(2B-32)+8\Rightarrow B=1$	M1	3.3
	$x=\mathrm{e}^{-t}(\sin 2t-8\cos 2t)+10$	A1	2.2a
	$y=\mathrm{e}^{-t}(2\sin 2t-3\cos 2t)+8$	A1	2.2a
		(4)	
(e)	E.g When t > 8, the amount of compound X and the amount of compound Y remain (approximately) constant at 10 and 8 respectively, which suggests that the chemical reaction has stopped. This supports the scientist's claim.	B1	3.5a
		(1)	
			(17 marks)

Question	Scheme	Marks	AOs
3(a)	$100m^2+60m+13=0 \Rightarrow m=-0.3\pm 0.2\mathrm{i}$	M1	1.1b
	$x=\mathrm{e}^{-0.3t}\left(A\cos 0.2t+B\sin 0.2t\right)$	A1	1.1b
	PI: $x = 2$	B1	1.1b
	$x=\mathrm{e}^{-0.3t}\left(A\cos 0.2t+B\sin 0.2t\right)+2$	A1ft	2.2a
		(4)	
(b)	$t=0,\ \ x=0 \Rightarrow A=-2$	M1	3.4
	$\frac{\mathrm{d}x}{\mathrm{d}t}=-0.3\mathrm{e}^{-0.3t}\left(-2\cos 0.2t+B\sin 0.2t\right)+\mathrm{e}^{-0.3t}\left(0.4\sin 0.2t+0.2B\cos 0.2t\right)$ $t=0,\ \frac{\mathrm{d}x}{\mathrm{d}t}=10 \Rightarrow B=\ldots$(NB B = 47)	M1	3.4
	$x=\mathrm{e}^{-0.3t}\left(47\sin 0.2t-2\cos 0.2t\right)+2$	A1	1.1b
	$-0.3\mathrm{e}^{-0.3t}\left(47\sin 0.2t-2\cos 0.2t\right)+\mathrm{e}^{-0.3t}\left(9.4\cos 0.2t+0.4\sin 0.2t\right)=0$ $\Rightarrow t=\ldots$ or $x=\sqrt{2213}\mathrm{e}^{-0.3t}\sin\left(0.2t-0.0425\right)+2$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=-0.3\sqrt{2213}\mathrm{e}^{-0.3t}\sin\left(0.2t-0.0425\right)$ $+0.2\sqrt{2213}\mathrm{e}^{-0.3t}\cos\left(0.2t-0.0425\right)$ $\Rightarrow t=\ldots$	M1	3.1b
	$\tan 0.2t=\frac{100}{137} \Rightarrow 0.2t=0.630\ldots$ or $\tan\left(0.2t-0.0425\right)=\frac{2}{3} \Rightarrow 0.2t=0.630$	M1	2.1
	$t = 3.15\ldots$ weeks	A1	1.1b
	$x=\mathrm{e}^{-0.3\times\text{"3.15..."}}\left(47\sin\left(0.2\times\text{"3.15..."}\right)-2\cos\left(0.2\times\text{"3.15..."}\right)\right)+2$	M1	3.4
	= awrt 12.1 {μg/ml}	A1	3.2a
		(8)	
(c)	$t=10 \Rightarrow x=\mathrm{e}^{-3}\left(47\sin(2)-2\cos(2)\right)+2=4.16\ldots$	M1	3.4
	The model suggests that it would be safe to give the second dose	A1ft	2.2a
		(2)	
			(14 marks)

Notes

(a)
M1: Uses the model to form and solve the auxiliary equation
A1: Correct CF, does not need x =
B1: Correct PI
A1ft: Deduces the correct GS (follow through their CF + PI). Must have $x=\mathrm{f}(t)$ and PI not 0

(b)
M1: Uses the model and the initial conditions to establish the value of "A"
M1: Differentiates their model using the product rule and uses the initial conditions to establish the value of "B". Must be using $x=0$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=10$
A1: Correct particular solution. This can be implied by the correct constants found following a correct answer to part (a).

Question	Scheme	Marks	AOs
6(a)	$5k(13.6)+2k(0)+17(-20)=0 \Rightarrow k=...$	M1	3.3
	$k=5$	A1	1.1b
		(2)	
(b)	Solves their $25m^2+10m+17=0 \Rightarrow m=...$	M1	3.1b
	$m=-0.2\pm 0.8\text{i}$	A1	1.1b
	$x=\text{e}^{-0.2t}\left(A\cos 0.8t+B\sin 0.8t\right)$	A1ft	1.1b
	$t=0,\ x=-20 \Rightarrow A=...\ (=-20)$	M1	3.4
	$\frac{\text{d}x}{\text{d}t}=-0.2\text{e}^{-0.2t}\left(A\cos 0.8t+B\sin 0.8t\right)$ $+\text{e}^{-0.2t}\left(-0.8A\sin 0.8t+0.8B\cos 0.8t\right)$	M1	1.1b
	$t=0\ \frac{\text{d}x}{\text{d}t}=0 \Rightarrow -0.2A+0.8B=0 \Rightarrow B=...\ (=-5)$	dM1	3.4
	$x=\text{e}^{-0.2t}\left(-20\cos 0.8t-5\sin 0.8t\right)$ o.e.	A1	1.1b
		(7)	
(c)	Vertical height $=30+\left[\text{e}^{-0.2\times 15}\left(-20\cos(0.8\times 15)-5\sin(0.8\times 15)\right)\right]$	M1	3.4
	Vertical height = awrt 29.3 m	A1	2.2b
		(2)	
(d)	For example It is unlikely that the rope will remain taut The model predicts the tourist will continue to move up and down, (but in fact they will lose momentum) The tourist is modelled as a particle	B1	3.5b
		(1)	
			(12 marks)

Notes:

(a)

M1: Substitutes $\frac{\text{d}^2x}{\text{d}t^2}=13.6$ $\frac{\text{d}x}{\text{d}t}=0$ and $x=-20$ into the differential equation to find a value for k.
Allow if there are sign slips but must be attempting the values in the correct places.
A1: Correct value $k=5$

(b)
M1: Forms and solves the auxiliary equation.
A1: Correct solution to the auxiliary equation (not follow through).
A1ft: Correct complementary function for their solutions to their auxiliary equation. (Follow through on distinct real, repeated or complex roots.)

uestion	Scheme		Marks	AOs
10(a)(i)	$\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	Let $\theta = \lambda t \sin 3t$ $\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	M1	1.1b
	$\frac{d\theta}{dt} = \frac{1}{12}\sin 3t + \frac{1}{4}t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \frac{1}{4}\cos 3t + \frac{1}{4}\cos 3t - \frac{3}{4}t \sin 3t$ $= \frac{1}{2}\cos 3t - \frac{3}{4}t \sin 3t$	$\frac{d\theta}{dt} = \lambda \sin 3t + 3\lambda t \cos 3t$ and $\frac{d^2\theta}{dt^2} = 3\lambda \cos 3t + 3\lambda \cos 3t - 9\lambda t \sin 3t$ $= 6\lambda \cos 3t - 9\lambda t \sin 3t$	A1	1.1b
	$\frac{1}{2}\cos 3t - \frac{3}{4}t \sin 3t + 9\left(\frac{1}{12}t \sin 3t\right) = \ldots$	$6\lambda \cos 3t - 9\lambda t \sin 3t + 9(\lambda t \sin 3t) = \frac{1}{2}\cos 3t \Rightarrow \lambda = \ldots$	dM1	3.4
	$= \frac{1}{2}\cos 3t$ so PI is $\theta = \frac{1}{12}t \sin 3t$ *	$\theta = \frac{1}{12}t \sin 3t$ *	A1*	2.1
			(4)	
(a)(ii)	$m^2 + 9 = 0 \Rightarrow m = \pm 3i$		M1	1.1b
	$\theta = A\cos 3t + B \sin 3t$		A1	1.1b
	$(\theta =) CF + PI$		dM1	1.1b
	$\theta = A\cos 3t + B\sin 3t + \frac{1}{12}t \sin 3t$		A1	1.1b
			(4)	
(b)	$t = 0,\ \theta = \frac{\pi}{3} \Rightarrow A = \ldots\left\{\frac{\pi}{3}\right\}$		M1	3.4
	$t = 0,\ \frac{d\theta}{dt} = -3A\sin 3t + 3B\cos 3t + \frac{1}{12}\sin 3t + \frac{1}{4}t\cos 3t = 0$ $\Rightarrow B = \ldots\{0\}$		M1	3.4
	$\alpha = \frac{\pi}{3}\cos(3\times 10) + \frac{1}{12}(10)\sin(3\times 10) = \ldots$		ddM1	1.1b
	$\alpha = \pm$awrt 0.662		A1	3.4
			(4)	
(c)	0.662 is close to 0.62 so a good model (at $t = 10$)		B1ft	3.5a
			(1)	
(d)	$\frac{d^2\theta}{dt^2} + 9\theta = 0$ oe		B1	3.5c
			(1)	

Question	Scheme	Marks	AOs
9(a)	$\frac{d^2y}{dt^2}=0.032\frac{dx}{dt}-0.025\frac{dy}{dt}$ $\quad$ $\frac{d^2y}{dt^2}=\frac{4}{125}\frac{dx}{dt}-\frac{1}{40}\frac{dy}{dt}$ $\frac{dx}{dt}=\frac{1}{0.032}\left(0.025\frac{dy}{dt}+\frac{d^2y}{dt^2}\right)$ $\quad$ $\frac{dx}{dt}=\frac{25}{32}\frac{dy}{dt}+\frac{125}{4}\frac{d^2y}{dt^2}$	**B1**	1.1b
	$\frac{d^2y}{dt^2}=0.032(0.025y-0.045x+2)-0.025\frac{dy}{dt}$ $\frac{d^2y}{dt^2}=0.0008y-0.00144x+0.064-0.025\frac{dy}{dt}$ $\frac{d^2y}{dt^2}=\frac{1}{1250}y-\frac{9}{6250}x+\frac{8}{125}-\frac{1}{40}\frac{dy}{dt}$ Then substitutes for x $\frac{d^2y}{dt^2}=0.0008y-\frac{0.00144}{0.032}\left(\frac{dy}{dt}+0.025y\right)+0.064-0.025\frac{dy}{dt}$ or $\frac{d^2y}{dt^2}=\frac{1}{1250}y-\frac{9}{6250}\left(\frac{125}{4}\frac{dy}{dt}+\frac{25}{32}y\right)+\frac{8}{125}-\frac{1}{40}\frac{dy}{dt}$ or $\frac{1}{0.032}\left(\frac{d^2y}{dt^2}+0.025\frac{dy}{dt}\right)=0.025y-\frac{0.045}{0.032}\left(\frac{dy}{dt}+0.025y\right)+2$	**M1**	1.1b
	$\left\{\frac{d^2y}{dt^2}=-0.000325y-0.07\frac{dy}{dt}+0.064\right\}$ $\left\{\frac{d^2y}{dt^2}=-\frac{13}{40000}y-\frac{7}{100}\frac{dy}{dt}+\frac{8}{125}\right\}$ $40\,000\frac{d^2y}{dt^2}+2800\frac{dy}{dt}+13y=2560$ *	**A1***	2.1
		(3)	
	Alternative $\frac{d^2y}{dt^2}=0.032\frac{dx}{dt}-0.025\frac{dy}{dt}$	**B1**	1.1b

	$40000\left[0.032\frac{dx}{dt}-0.025\frac{dy}{dt}\right]+2800\left[0.032x-0.025y\right]+\frac{13}{0.025}\left[0.032x-\frac{dy}{dt}\right]$ $=1280\frac{dx}{dt}-1000\frac{dy}{dt}+89.6x-70y+16.64x-520\frac{dy}{dt}$ $=1280\left[0.025y-0.045x+2\right]-1520\left[0.032x-0.025y\right]$ $=A$	**M1**	1.1b
	$32y-57.6x+2560-48.64x+38y-70y+106.24=2560*$	**A1***	2.1
		(3)	
(b)	$40000m^2+2800m+13\{=0\}\Rightarrow m=...$	**M1**	3.4
	$\text{CF}: y=Ae^{m_1t}+Be^{m_2t}$	**M1**	1.1b
	$\text{CF}: y=Ae^{\frac{-t}{200}}+Be^{\frac{-13t}{200}}$ $\text{CF}: y=Ae^{-0.005t}+Be^{-0.065t}$	**A1**	1.1b
	PI: Try $y=k\Rightarrow 13k=2560\Rightarrow k=...\left\{\frac{2560}{13}\right\}$	**M1**	3.4
	GS: $y=Ae^{\frac{-t}{200}}+Be^{\frac{-13t}{200}}+\frac{2560}{13}$ $GS: y=Ae^{-0.005t}+Be^{-0.065t}+\frac{2560}{13}$	**A1ft**	1.1b
		(5)	
(c)	$t=0, y=0\Rightarrow 0=A+B+\frac{2560}{13}$	**M1**	3.4
	$t=0, y=0, x=0\Rightarrow\frac{dy}{dt}=0.032\times0-0.025\times0=0$ Or Used $x=\frac{1}{0.032}\left(\frac{dy}{dt}+0.025y\right)$ to find an equation in t	**B1**	3.4
	$\Rightarrow\frac{dy}{dt}=-\frac{A}{200}e^{\frac{-t}{200}}-\frac{13B}{200}e^{\frac{-13t}{200}}=0\Rightarrow-\frac{A}{200}-\frac{13B}{200}=0\Rightarrow A=-13B$ Or $x=\frac{1}{0.032}\left[-0.005Ae^{\frac{-t}{200}}-0.065Be^{\frac{-13t}{200}}+0.025\left(Ae^{\frac{-t}{200}}+Be^{\frac{-13t}{200}}+\frac{2560}{13}\right)\right]$	**M1**	1.1b

	$x=\frac{5}{8}Ae^{\frac{-t}{200}}-\frac{5}{4}Be^{\frac{-13t}{200}}+\frac{2000}{13}\Rightarrow 0=\frac{5}{8}A-\frac{5}{4}B+\frac{2000}{13}$		
	$y=-\frac{640}{3}e^{\frac{-t}{200}}+\frac{640}{39}e^{\frac{-13t}{200}}+\frac{2560}{13}$	**A1**	1.1b
		(4)	
(d)	As $t\to\infty, e^{-kt}\to 0$ for $k>0$ so $y\to\ldots$,	**M1**	1.1b
	$y\to\frac{2560}{13}\approx 196$ or 197 so the rate of administration is sufficient to reach the required level.	**A1ft**	3.2b
		(2)	
			(14 marks)

Notes:

(a)

B1: Differentiates the second equation with respect to *t* correctly. May have rearranged to make *x* the subject first. The dot notation for derivatives may be used.

M1: Uses the second equation to eliminate *x* to achieve an equation in *y*, $\frac{dy}{dt}, \frac{d^2y}{dt^2}$.

A1*: Achieves the printed answer with no errors, allow dot notation

$$40\,000\ddot{y}+2800\dot{y}+13y=2560$$

Alternative

B1: Differentiates the second equation with respect to *t* correctly. May have rearranged to make *x* the subject first. The dot notation for derivatives may be used.

M1: Substitutes in the printed differential equation uses both equations to remove all derivative, form an expression involving *x*'s and *y*'s which simplifies to a constant.

A1*: Achieves 2560 with no errors seen

(b)

M1: Uses the model to form and attempt to solve the auxiliary equation the PI (Accept a correct equation followed by two values for *m* as an attempt to solve.)

M1: Forms the complementary function correct for their roots (so if repeated or complex roots found, award for appropriate form for CF). Must be in terms of *t* only (not *x*)

A1: Correct CF