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Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Core Mathematics 4 (6666/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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Question Number	Scheme	Notes	Marks
1. Way 1	$\left\{\dfrac{1}{(2+5x)^3} = \right\} (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$= \underline{(2)^{-3}}\left(1+\dfrac{5x}{2}\right)^{-3} = \underline{\dfrac{1}{8}}\left(1+\dfrac{5x}{2}\right)^{-3}$	$\underline{2^{-3}}$ or $\underline{\dfrac{1}{8}}$	$\underline{\text{B1}}$
	$= \left\{\dfrac{1}{8}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$= \left\{\dfrac{1}{8}\right\}\left[1+(-3)\left(\dfrac{5x}{2}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{5x}{2}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{5x}{2}\right)^3+\ldots\right]$		
	$= \dfrac{1}{8}\left[1-\dfrac{15}{2}x+\dfrac{75}{2}x^2-\dfrac{625}{4}x^3+\ldots\right]$		
	$= \dfrac{1}{8}\left[1-7.5x+37.5x^2-156.25x^3+\ldots\right]$		
	$= \dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$ **or** $\dfrac{1}{8}-\dfrac{15}{16}x;+4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3+\ldots$		A1; A1
			[6]
			6
Way 2	$\mathrm{f}(x)=(2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$\mathrm{f}''(x)=300(2+5x)^{-5},\ \mathrm{f}'''(x)=-7500(2+5x)^{-6}$	Correct $\mathrm{f}''(x)$ and $\mathrm{f}'''(x)$	B1
	$\mathrm{f}'(x)=-15(2+5x)^{-4}$	$\pm a(2+5x)^{-4},\ a\neq\pm1$	M1
		$-15(2+5x)^{-4}$	A1 oe
	$\left\{\therefore \mathrm{f}(0)=\dfrac{1}{8},\ \mathrm{f}'(0)=-\dfrac{15}{16},\ \mathrm{f}''(0)=\dfrac{75}{8}\ \text{and}\ \mathrm{f}'''(0)=-\dfrac{1875}{16}\right\}$		
	So, $\mathrm{f}(x)=\dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
			[6]
Way 3	$(2+5x)^{-3}$	Same as in Way 1	M1
	$= \underline{(2)^{-3}}+(-3)(2)^{-4}(5x)+\dfrac{(-3)(-4)}{2!}(2)^{-5}(5x)^2+\dfrac{(-3)(-4)(-5)}{3!}(2)^{-6}(5x)^3$	Same as in Way 1	$\underline{\text{B1}}$
		Any two terms correct	M1
		All four terms correct	A1
	$= \dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$	Same as in Way 1	A1; A1
	Note: Terms can be simplified or un-simplified for B1 2nd M1 1st A1		**[6]**
	Note: The terms in C need to be evaluated So ${}^{-3}C_0(2)^{-3}+{}^{-3}C_1(2)^{-4}(5x)+{}^{-3}C_2(2)^{-5}(5x)^2+{}^{-3}C_3(2)^{-6}(5x)^3$ without further working is B0 1st M0 1st A0		

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		Question 1 Notes
1.	**1st M1**	mark can be implied by a constant term of $(2)^{-3}$ or $\frac{1}{8}$.
	B1	$\underline{2^{-3}}$ or $\underline{\frac{1}{8}}$ outside brackets or $\frac{1}{8}$ as candidate's constant term in their binomial expansion.
	2nd M1	Expands $(\dots + kx)^{-3}$, $k =$ a value $\neq 1$, to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+(-3)(kx)$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\ \dots\ +\frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.
	1st A1	A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent and $k =$ a value $\neq 1$. (on the RHS, not necessarily the LHS) in a candidate's expansion.
	Note	You would award B1M1A0 for $\frac{1}{8}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}(5x)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3+\dots\right]$ because (kx) is not consistent.
	Note	**Incorrect bracketing:** $=\left\{\frac{1}{8}\right\}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{5x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x^3}{2}\right)+\dots\right]$ is M1A0 unless recovered.
	2nd A1	For $\frac{1}{8}-\frac{15}{16}x$ **(simplified)** or also allow $0.125 - 0.9375x$.
	3rd A1	Accept only $\frac{75}{16}x^2 - \frac{625}{32}x^3$ or $4\frac{11}{16}x^2 - 19\frac{17}{32}x^3$ or $4.6875x^2 - 19.53125x^3$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either**
		SC: $\frac{1}{8}\left[1-\frac{15}{2}x\ ;\ \dots\right]$ or **SC:** $\frac{1}{8}\left[1+\dots+\frac{75}{2}x^2+\dots\right]$ or **SC:** $\frac{1}{8}\left[1+\dots-\frac{625}{4}x^3+\dots\right]$
		SC: $\lambda\left[1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+\dots\right]$ or **SC:** $\left[\lambda-\frac{15\lambda}{2}x+\frac{75\lambda}{2}x^2-\frac{625\lambda}{4}x^3+\dots\right]$
		(where λ can be 1 or omitted), where each term in the $[\dots\dots]$ is a simplified fraction or a decimal
	SC	<u>**Special case for the 2nd M1 mark**</u> Award Special Case 2nd M1 for a correct simplified or un-simplified $1+n(kx)+\frac{n(n-1)}{2!}(kx)^2+\frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n \neq -3$, $n \neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$ **.**
	Note	Ignore extra terms beyond the term in x^3
	Note	You can ignore subsequent working following a correct answer.

Question Number	Scheme		Marks
2.	x: 1, 1.2, 1.4, 1.6, 1.8, 2; y: 0, 0.2625, ***0.659485...***, 1.2032, 1.9044, 2.7726	$y = x^2 \ln x$	
(a)	$\{\text{At } x = 1.4,\}\ y = 0.6595\ (4\text{ dp})$	0.6595	B1 **cao**
			[1]
(b)	$\frac{1}{2} \times (0.2) \times \underline{\left[0 + 2.7726 + 2\left(0.2625 + \text{ their } 0.6595 + 1.2032 + 1.9044\right)\right]}$	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$	B1 o.e.
	{**Note:** The "0" does not have to be included in [……]}	<u>For structure of</u> $\underline{[\ldots\ldots\ldots\ldots\ldots]}$	M1
	$\left\{= \frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083\ (3\text{ dp})$	anything that rounds to 1.083	A1
			[3]
(c) **Way 1**	$\left\{\text{I} = \int x^2 \ln x \,\text{d}x\right\}$, $\left\{\begin{matrix} u = \ln x \Rightarrow \frac{\text{d}u}{\text{d}x} = \frac{1}{x} \\ \frac{\text{d}v}{\text{d}x} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{matrix}\right\}$		
	$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\text{d}x\}$	Either $x^2 \ln x \to \pm\lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{\text{d}x\}$ or $\pm\lambda x^3 \ln x - \int \mu x^2 \{\text{d}x\}$, where $\lambda, \mu > 0$	M1
		$x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\text{d}x\}$, simplified or un-simplified	A1
	$= \frac{x^3}{3}\ln x - \frac{x^3}{9}$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
	$\text{Area}(R) = \left\{\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2\right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$	**dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round	dM1
	$= \frac{8}{3}\ln 2 - \frac{7}{9}$	$\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$	A1 oe **cso**
			[5]
(c) **Way 2**	$\text{I} = x^2(x\ln x - x) - \int 2x(x\ln x - x)\text{d}x$	$\left\{\begin{matrix} u = x^2 \Rightarrow \frac{\text{d}u}{\text{d}x} = 2x \\ \frac{\text{d}v}{\text{d}x} = \ln x \Rightarrow v = x\ln x - x \end{matrix}\right\}$	
	So, $3\text{I} = x^2(x\ln x - x) + \int 2x^2 \{\text{d}x\}$		
	and $\text{I} = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{\text{d}x\}$	A full method of applying $u = x^2,\ v' = \ln x$ to give $\pm\lambda x^2(x\ln x - x) \pm \mu \int x^2 \{\text{d}x\}$	M1
		$\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{\text{d}x\}$ simplified or un-simplified	A1
	$= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]
			9

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		Question 2 Notes
2. (a)	**B1**	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	M1	For structure of trapezium rule $[\ \ldots\ldots\ldots\ldots\ ]$
	Note	No errors are allowed [eg. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y* ordinate].
	A1	anything that rounds to 1.083
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…)
	Note	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	Note	Award B1M1A1 for $\frac{1}{10}(2.7726)+\frac{1}{5}(0.2625+\text{their } 0.6595+1.2032+1.9044)=$ awrt 1.083
		Bracketing mistake: Unless the final answer implies that the calculation has been done correctly
		Award B1M0A0 for $\frac{1}{2}(0.2)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)+2.7726$ (answer of 10.9318)
		Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)$ (answer of 8.33646)
		Alternative method: Adding individual trapezia
		Area $\approx 0.2\times\left[\frac{0+0.2625}{2}+\frac{0.2625+"0.6595"}{2}+\frac{"0.6595"+1.2032}{2}+\frac{1.2032+1.9044}{2}+\frac{1.9044+2.7726}{2}\right]=1.08318...$
	B1	0.2 and a divisor of 2 on all terms inside brackets
	M1	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2
	A1	anything that rounds to 1.083
(c)	A1	Exact answer needs to be a two term expression in the form $a\ln b+c$
	Note	Give A1 e.g. $\frac{8}{3}\ln 2-\frac{7}{9}$ or $\frac{1}{9}(24\ln 2-7)$ or $\frac{4}{3}\ln 4-\frac{7}{9}$ or $\frac{1}{3}\ln 256-\frac{7}{9}$ or $-\frac{7}{9}+\frac{8}{3}\ln 2$ or $\ln 2^{\frac{8}{3}}-\frac{7}{9}$ or equivalent.
	Note	Give final A0 for a final answer of $\frac{8\ln 2-\ln 1}{3}-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{1}{3}\ln 1-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{8}{9}+\frac{1}{9}$ or $\frac{8}{3}\ln 2-\frac{7}{9}+c$
	Note	$\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	Note	Give dM0A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\frac{1}{9}$ (adding rather than subtracting)
	Note	Allow dM1A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\left(0+\frac{1}{9}\right)$
	SC	A candidate who uses $u=\ln x$ and $\frac{dv}{dx}=x^2$, $\frac{du}{dx}=\frac{\alpha}{x}$, $v=\beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1.

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Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) **Way 1**	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \times\right\}$ $\underline{\underline{\left(4xy + 2x^2\frac{dy}{dx}\right)}}$ $\underline{+ 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0}$		M1 <u>A1</u> <u>B1</u>
	$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{dy}{dx} = \right\}\ \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)}$ $\left\{= \frac{-8}{22 + \pi}\right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\}$ $x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \times\right\}$ $\underline{\underline{\left(4xy\frac{dx}{dy} + 2x^2\right)}}$ $\underline{+ 2\frac{dx}{dy} + 4 + \pi\sin(\pi y) = 0}$		M1 <u>A1</u> <u>B1</u>
	$\frac{dx}{dy}\left(4xy + 2\right) + 2x^2 + 4 + \pi\sin(\pi y) = 0$		dM1
	$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]

Question 3 Notes

3. (a)	**Note**	Writing down ***from no working*** • $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi\sin(\pi y)}$ scores M1A0B1M1A0
	Note	Few candidates will write $4xy\,dx + 2x^2dy + 2dx + 4dy + \pi\sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or equivalent. This should get full marks.

		Question 3 Notes Continued
3. (a) **Way 1**	**M1**	Differentiates implicitly to include either $2x^2\frac{dy}{dx}$ or $4y \to 4\frac{dy}{dx}$ or $-\cos(\pi y) \to \pm\lambda\sin(\pi y)\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1	$2x + 4y - \cos(\pi y) = 17 \;\to\; 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$
	Note	$4xy + 2x^2\frac{dy}{dx} + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} \;\to\; 2x^2\frac{dy}{dx} + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = -4xy - 2$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	B1	$2x^2y \to 4xy + 2x^2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	dM1	**Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. ie. $\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + \;...\; = \;...$
	Note	Writing down an extra $\frac{dy}{dx} = ...$ and then including it in their factorisation is fine for dM1.
	Note	**Final A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from correct solution.
(a)	**Way 2**	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	**1st M1**	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$. E.g. $"-4xy" \to "-6"$ in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}$, $y = 3$.
	3rd M1	***is dependent on the first M1.***
	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 2nd M1 3rd M1** for $\frac{\frac{1}{2}}{3-x} = \frac{-1}{\text{their } m_T}$
	Note	We can accept $\sin\pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2nd M1 mark. But, $\sin\pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3rd M1 mark. The 3rd M1 can be accessed for terms containing $\pi\sin\left(\frac{\pi}{2}\right)$.

Question Number	Scheme	Notes	Marks
4.	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $\ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow$ $\underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 2**	$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,\mathrm{d}x$	**Either** $\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ **or** $t = \int -\frac{2}{5x}\,\mathrm{d}x$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = ...$ **or** $\pm\alpha\ln px$; $\alpha \neq 0$, $p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow$ $\underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 3**	$\int_{60}^{x} \frac{1}{x}\,\mathrm{d}x = \int_{0}^{t} -\frac{5}{2}\,\mathrm{d}t$	Ignore limits	B1
	$\left[\ln x\right]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\left[\ln x\right]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow$ $\underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]
(b)	$20 = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda\mathrm{e}^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda\mathrm{e}^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449...\text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$ ⎕ , $t > 0$)	dM1
	$\Rightarrow t = 632.8006... = 633$(to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 **cso**
	Note: dM1 can be implied by $t = $ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
4.	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\int \frac{2}{5x}\,\mathrm{d}x = -\int \mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 5**	$\left\{\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x} \Rightarrow\right\}\quad t = \int_{60}^{x} -\frac{2}{5x}\,\mathrm{d}x$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]

		Question 4 Notes
4. (a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,\mathrm{d}x = \int -\frac{1}{2}\,\mathrm{d}t$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{\mathrm{e}^{-5t}}$ or $x = \frac{60}{\sqrt{\mathrm{e}^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = \mathrm{e}^{-\frac{5}{2}t} + 60 \to x = 60\mathrm{e}^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = \mathrm{e}^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60\mathrm{e}^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

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Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right)$, $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = \text{awrt } 8.7$	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

	Question 5 Notes	
5. (a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin\left(2(45)\right)$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\quad 0 \leqslant t < \dfrac{\pi}{2}$		
(a) **Way 2**	$\tan t = \dfrac{x}{4} \Rightarrow \sin t = \dfrac{x}{\sqrt{(x^2+16)}},\ \cos t = \dfrac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \dfrac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{array}{ll} u = 40\sqrt{3}x & v = x^2+16 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = 40\sqrt{3} & \dfrac{\mathrm{d}v}{\mathrm{d}x} = 2x \end{array}\right\}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2}\ \left\{= \dfrac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\dfrac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; simplified or un-simplified	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$	$-\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)\left(\dfrac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\dfrac{1}{4}\right)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm A\cos\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)\left(\dfrac{1}{1+x^2}\right)$	M1
		Correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; simplified or un-simplified.	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\dfrac{2}{1+3}\right)\left(\dfrac{1}{4}\right)\ \left\{= 5\sqrt{3}\left(-\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$	$-\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,\ x = 4\sin^2\theta$		
(i) **Way 1**	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow -4 = 2A \Rightarrow A = -2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y = -\frac{2}{3} \Rightarrow -6 = -\frac{2}{3}B \Rightarrow B = 9$	Both their $A=-2$ **and** their $B=9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y} + \frac{9}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $A\neq 0,\ B\neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]
(ii) (a) **Way 1**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 8\sin\theta\cos\theta\,\mathrm{d}\theta$		B1
	$\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,4\sin 2\theta\,\{\mathrm{d}\theta\}$		M1
	$= \int \underline{\underline{\tan\theta}}\,.\,8\sin\theta\cos\theta\,\{\mathrm{d}\theta\}$ or $\int \underline{\underline{\tan\theta}}\,.\,4\sin 2\theta\,\{\mathrm{d}\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1
	$3 = 4\sin^2\theta$ **or** $\frac{3}{4} = \sin^2\theta$ **or** $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$; $\{x=0 \to \theta = 0\}$	Writes down a correct equation involving $x=3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$= \{8\}\int\left(\frac{1-\cos 2\theta}{2}\right)\mathrm{d}\theta\quad \left\{= \int(4-4\cos 2\theta)\,\mathrm{d}\theta\right\}$	Applies $\cos 2\theta = 1-2\sin^2\theta$ to their integral. (See notes)	M1
	$= \{8\}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\quad \{= 4\theta - 2\sin 2\theta\}$	For $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha,\beta\neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0+0)\right)$		
	$= \frac{4}{3}\pi - \sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$	A1 o.e.
			[4]
			15

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	Question 6 Notes	
6. (i)	**1st M1**	Writing $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y} + \dfrac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied ***for writing down*** either $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{-2}{y} + \dfrac{\text{their } B}{(3y+2)}$ or $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{\text{their } A}{y} + \dfrac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\dfrac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\dfrac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\dfrac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
6. (ii)(a)	**1st M1** **Note** **Note**	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{\left(\dfrac{x}{4-x}\right)}\,\mathrm{d}x$ $\mathrm{d}x \neq \lambda\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$ Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1 **Note**	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\dfrac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\dfrac{\sin\theta}{\cos\theta}\right)$ Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \dfrac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\dfrac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\dfrac{x}{4}}\right)$ followed by $x = 3,\ \theta = \dfrac{\pi}{3};\ x = 0,\ \theta = 0$
(ii)(b)	**M1**	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\dfrac{1-\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0,\ \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\dfrac{1}{2}\theta - \dfrac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\dfrac{k}{2}\theta - \dfrac{k}{4}\sin 2\theta$ or $\dfrac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A ***correct solution in part (ii)*** leading to a "two term" exact answer of e.g. $\dfrac{4}{3}\pi - \sqrt{3}$ or $\dfrac{8}{6}\pi - \sqrt{3}$ or $\dfrac{4}{3}\pi - \dfrac{2\sqrt{3}}{2}$ or $\dfrac{1}{3}\left(4\pi - 3\sqrt{3}\right)$
	Note	A decimal answer of 2.456739397... (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
6. (i) **Way 2**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{6y+2}{3y^2+2y}\,\mathrm{d}y - \int \frac{3y+6}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{6y+2}{3y^2+2y}\,\mathrm{d}y - \int \frac{3}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]
6. (i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y+1}{3y^2+2y}\,\mathrm{d}y - \int \frac{5}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3y+1}{3y^2+2y}\,\mathrm{d}y - \int \frac{\frac{5}{2}}{y}\,\mathrm{d}y + \int \frac{\frac{15}{2}}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]

	Scheme	Notes	
6. (i) **Way 4**	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$= \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $= \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0,\ B \neq 0,\ C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]
	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) **Way 2**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$	As before	M1
	$= \int \sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sin\theta\,.\,8\sin\theta\ \{\mathrm{d}\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 **cso**
(ii)(a) **Way 3**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta,\quad 4 - x = 2 + 2\cos 2\theta$		
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		M1
	$= \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}4\sin 2\theta\ \{\mathrm{d}\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		
	$= \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int 2(2-2\cos 2\theta).\ \{\mathrm{d}\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 **cso**

Question Number	Scheme	Notes	Marks
7.	$y=(2x-1)^{\frac{3}{4}}\,,\quad x\geqslant\frac{1}{2}$ passes though $P(k\,,8)$		
(a)	$\left\{\int(2x-1)^{\frac{3}{2}}\,\mathrm{d}x\right\}=\frac{1}{5}(2x-1)^{\frac{5}{2}}\,\{+c\}$	$(2x\pm1)^{\frac{3}{2}}\to\pm\lambda(2x\pm1)^{\frac{5}{2}}$ **or** $\pm\lambda u^{\frac{5}{2}}$ where $u=2x\pm1$; $\lambda\neq0$	M1
		$\frac{1}{5}(2x-1)^{\frac{5}{2}}$ with or without $+\,c$. Must be simplified.	A1
			[2]
(b)	$\{P(k\,,8)\Rightarrow\}\ 8=(2k-1)^{\frac{3}{4}}\Rightarrow k=\dfrac{8^{\frac{4}{3}}+1}{2}$	Sets $8=(2k-1)^{\frac{3}{4}}$ or $8=(2x-1)^{\frac{3}{4}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k=\dfrac{17}{2}$	k (or x) $=\dfrac{17}{2}$ or 8.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2\mathrm{d}x$	For $\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2$ or $\pi\int(2x-1)^{\frac{3}{2}}$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}}y^2\,\mathrm{d}x\right\}=\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}}=\left(\left(\dfrac{16^{\frac{5}{2}}}{5}\right)-(0)\right)\ \left\{=\dfrac{1024}{5}\right\}$ **Note:** It is not necessary to write the "-0"	Applies x-limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^{\frac{5}{2}}$; $\beta\neq0$ and subtracts the correct way round.	M1
	$\left\{V_{\text{cylinder}}\right\}=\pi(8)^2\left(\dfrac{17}{2}\right)\ \{=544\pi\}$	$\pi(8)^2$(their answer to part (b)) $V_{\text{cylinder}}=544\pi$ implies this mark	B1 ft
	$\left\{\text{Vol}(S)=544\pi-\dfrac{1024\pi}{5}\right\}\Rightarrow\text{Vol}(S)=\dfrac{1696}{5}\pi$	An exact correct answer in the form $k\pi$ E.g. $\dfrac{1696}{5}\pi,\ \dfrac{3392}{10}\pi$ or 339.2π	A1
			[4]
Alt. (c)	$\text{Vol}(S)=\pi(8)^2\left(\dfrac{1}{2}\right)+\underline{\underline{\pi}}\int_{0.5}^{8.5}\left(8^2-\underline{\underline{(2x-1)^{\frac{3}{2}}}}\right)\mathrm{d}x$	For $\underline{\underline{\pi}}\int\ldots..\ \underline{\underline{(2x-1)^{\frac{3}{2}}}}$ Ignore limits and $\mathrm{d}x$.	B1
	$=\pi(8)^2\left(\dfrac{1}{2}\right)+\pi\left[64x-\dfrac{1}{5}(2x-1)^{\frac{5}{2}}\right]_{0.5}^{8.5}$		
	$=\underline{\underline{\pi(8)^2\left(\dfrac{1}{2}\right)}}+\underline{\underline{\pi}}\left(\left(\underline{\underline{64(\text{"}8.5\text{"})}}-\dfrac{1}{5}(2(8.5)-1)^{\frac{5}{2}}\right)-\left(\underline{\underline{64(0.5)}}-\dfrac{1}{5}(2(0.5)-1)^{\frac{5}{2}}\right)\right)$	as above	M1 $\underline{\underline{\text{B1}}}$
	$\left\{=32\pi+\pi\left(\left(544-\dfrac{1024}{5}\right)-(32-0)\right)\right\}\Rightarrow\text{Vol}(S)=\dfrac{1696}{5}\pi$		A1
			[4]
			8

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		Question 7 Notes
7. (b)	**SC**	Allow Special Case SC M1 for a candidate who sets $8 = (2k-1)^{\frac{3}{2}}$ or $8 = (2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.
7. (c)	**M1**	Can also be given for applying *u*-limits of "16" $(2(\text{"part }(b)\text{"}) - 1)$ and 0 to an expression of the form $\pm\beta u^{\frac{5}{2}}$; $\beta \neq 0$ and subtracts the correct way round.
	Note	You can give M1 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}} = \dfrac{1024}{5}$
	Note	Give M0 for $\left[\dfrac{(2x-1)^{\frac{5}{2}}}{5}\right]_{0}^{\frac{17}{2}} = \left(\left(\dfrac{16^{\frac{5}{2}}}{5}\right) - (0)\right)$
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height *k*.
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi\int_0^{8.5} 8^2\,\mathrm{d}x = \pi\left[64x\right]_0^{8.5}$ is **not sufficient** for B1 but $\pi(64(8.5) - 0)$ **is sufficient** for B1.

7.	**MISREADING IN BOTH PARTS (B) AND (C)**		
	Apply the misread rule (MR) for candidates who apply $y = (2x-1)^{\frac{3}{2}}$ to **both** parts (b) and (c)		
(b)	$\{P(k,8) \Rightarrow\}\ 8 = (2k-1)^{\frac{3}{2}} \Rightarrow k = \dfrac{8^{\frac{2}{3}}+1}{2}$	Sets $8 = (2k-1)^{\frac{3}{2}}$ or $8 = (2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k = \dfrac{5}{2}$	k (or x) $= \dfrac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2 \mathrm{d}x$	For $\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2$ or $\pi\int(2x-1)^3$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}} y^2\,\mathrm{d}x\right\} = \left[\dfrac{(2x-1)^4}{8}\right]_{\frac{1}{2}}^{\frac{5}{2}} = \left(\left(\dfrac{4^4}{8}\right) - (0)\right)\ \{= 32\}$	Applies *x*-limits of "2.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^4$; $\beta \neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}} = \pi(8)^2\left(\dfrac{5}{2}\right)\ \{= 160\pi\}$	$\pi(8)^2$(their answer to part (b)) Sight of 160π implies this mark	B1 ft
	$\{\text{Vol}(S) = 160\pi - 32\pi\} \Rightarrow \text{Vol}(S) = 128\pi$	An exact correct answer in the form $k\pi$ E.g. 128π	A1
			[4]
	Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0	
	Note	If a candidate uses $y = (2x-1)^{\frac{3}{4}}$ in part (b) and then uses $y = (2x-1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).	

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Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu = 1$. $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$		
(a)	$A(3, 5, 0)$	$(3, 5, 0)$	B1
			[1]
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$	A1
	$\mathbf{d}_2$ is the direction vector of l_2	Do not allow $l_2 :$ **or** $l_2 \rightarrow$ **or** $l_1 =$ for the A1 mark.	**[2]**
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding AP	M1
		$2\sqrt{2}$	A1
			[2]
(d)	So $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	Realisation that the dot product is required between $\left(\overrightarrow{AP} \text{ or } \overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AP} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2} . \sqrt{(-5)^2 + (4)^2 + (3)^2}}$	**dependent on the previous M mark.** Applies dot product formula between their $\left(\overrightarrow{AP} \text{ or } \overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	dM1
	$\{\cos\theta\} = \dfrac{\pm(10 + 0 + 6)}{\sqrt{8}.\sqrt{50}} = \dfrac{4}{5}$	$\{\cos\theta\} = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$	A1 cso
			[3]
(e)	$\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$	$\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ or $\dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$	M1
	$= 2.4$	2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40	A1
			[2]
(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c)		
	$\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \dfrac{4}{25} \Rightarrow\right\}\ \lambda = \pm\dfrac{2}{5}$	Either $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \dfrac{2}{5}\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$	**dependent on the previous M mark** Substitutes at least one of their values of λ into l_2.	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix} 3 \\ \frac{17}{5} \\ \frac{4}{5} \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix} -1 \\ \frac{33}{5} \\ \frac{16}{5} \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5]
			15

	Question 8 Notes	
8. (a)	**B1**	Allow $A(3, 5, 0)$ or $3\mathbf{i}+5\mathbf{j}$ or $3\mathbf{i}+5\mathbf{j}+0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$
(b)	**A1**	Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 = i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.
	Note	Allow the use of parameters μ or t instead of λ.
(c)	**M1**	Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find AP
	Note	Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2+(0)^2+(2)^2} = \sqrt{8} = 2\sqrt{2}$.
(d)	**Note**	For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a).
	Note	Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1
	Note	***Evaluating*** the dot product (i.e. $(-2)(-5)+(0)(4)+(2)(3)$) is not required for M1 and dM1 marks.
	Note	In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$
	Note	$\cos\theta = \dfrac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\dfrac{4}{5}$ followed by $\cos\theta = \dfrac{4}{5}$ is fine for A1 cso
	Note	Give M1dM1A1 for $\{\cos\theta =\} = \dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \dfrac{20+12}{40} = \dfrac{4}{5}$
	Note	Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^\circ$
	Alternative Method: Vector Cross Product	
	Only apply this scheme if it is clear that a candidate is applying a vector cross product method.	
	$\overrightarrow{AP}\times\mathbf{d}_2 = \begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-2&0&2\\-5&4&3\end{vmatrix} = -8\mathbf{i}-4\mathbf{j}-8\mathbf{k}\right\}$	Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — M1
	$\sin\theta = \dfrac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$	Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ — dM1
	$\sin\theta = \dfrac{12}{\sqrt{8}.\sqrt{50}} = \dfrac{3}{5} \Rightarrow \underline{\cos\theta = \dfrac{4}{5}}$	$\cos\theta = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ — A1
(e)	**Note**	Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^\circ - 36.869...^\circ)$; = awrt 2.40
	Note	Candidates must use their θ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$

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		Question 8 Notes Continued	
8. (f)	**Note**	Allow the first M1A1 for deducing $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ from no incorrect working	
	SC	Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working	
	Note	Give 1st M1 for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$	
	Note	Give 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent	
	Note	Give 1st M1 for $\lambda = \frac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$	
	Note	So $\left\{\hat{\mathbf{d}}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ "vector" $= \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1	
	Note	The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct *x*, *y*, *z* ordinates from their values of λ.	
	Note	Giving their "coordinates" as a column vector or position vector is fine for the final A1A1.	
	CAREFUL	Putting l_2 equal to *A* gives $\begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}3\\5\\0\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = \frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$	Give M0 dM0 for finding and using $\lambda = \frac{2}{5}$ from this incorrect method.
	CAREFUL	Putting $\lambda\mathbf{d}_2 = \overrightarrow{AP}$ gives $\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix} = \begin{pmatrix}2\\0\\-2\end{pmatrix} \rightarrow \begin{pmatrix}\lambda = -\frac{2}{5}\\ \lambda = 0\\ \lambda = -\frac{2}{3}\end{pmatrix}$	Give M0 dM0 for finding and using $\lambda = -\frac{2}{5}$ from this incorrect method.
	General	You can follow through the part (c) answer of their $AP = 2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1	
	General	You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1.	

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