

Mark Scheme (Final)

Summer 2015

Pearson Edexcel International A Level
in Core Mathematics C12 (WMA01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - \square or d... The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	The line l_1 has equation $10x - 2y + 7 = 0$	
(a)	Gradient is 5	B1 [1]
(b)	Gradient of parallel line is equal to their previous gradient Equation is $y - \frac{4}{3} = 5\left(x - \left(-\frac{1}{3}\right)\right)$ So $y = 5x + 3$	M1 M1 A1 [3] (4 marks)

- (a)
B1 Gradient given as 5 or $10/2$ or exact equivalent. Do not accept if embedded within an equation. You must see 5 (or equivalent)
- (b)
M1 Gradient of lines are the same. This may be implied by sight of their '5' in a gradient equation. For example you may see $y = 5x + c$ or equivalent as the equation of their "parallel" line .
- M1 For an attempt to find an equation of a line using $\left(-\frac{1}{3}, \frac{4}{3}\right)$ and a numerical gradient (which may be different to the gradient used in part (a). For example they may try to find a normal!) It must be a full attempt to find an equation.
Accept $y - \frac{4}{3} = \text{"numerical } m\text{"}\left(x - -\frac{1}{3}\right)$ or equivalent. Allow one sign slip for the coordinate.
If $y = mx + c$ is used it must proceed as far as finding the value of "c"
- A1 Correct answer only (no equivalents) $y = 5x + 3$, but do allow $y = 5x + c$ followed by $c = 3$.

Question Number	Scheme	Marks
2.	$f(x) = x^4 - x^3 + 3x^2 + ax + b$ Attempts to set $f(\pm 1) = 4$ or attempts to set $f(\pm 2) = 22$ Obtains $3 + a + b = 4$ or equivalent Obtains $16 + 8 + 12 - 2a + b = 22$ or equivalent Solve simultaneous equations to obtain $a = 5$ and $b = -4$	M1 A1 A1 M1 A1 [5] (5 marks)

M1 Attempts to set either $f(\pm 1) = 4$ or $f(\pm 2) = 22$
 Alternatively dividing $f(x)$ by $(x - 1)$ or dividing $f(x)$ by $(x + 2)$ and setting the remainder equal to 4 or 22
 For division look for a minimum of

$$x-1 \overline{)x^4 - x^3 + 3x^2 + ax + b} \quad \text{OR} \quad x+2 \overline{)x^4 - x^3 + 3x^2 + ax + b}$$

$$\overline{(\dots a) + (\dots b)}$$

$$\overline{(\dots a) + (\dots b)}$$

followed by the remainder (involving both a and b) set equal to 4 or 22

A1 $3 + a + b = 4$ or equivalent.

The powers must be multiplied out but there is no requirement to collect terms.

A1 $16 + 8 + 12 - 2a + b = 22$

The powers must be multiplied out but there is no requirement to collect terms.

M1 Solves a pair of simultaneous equations in both a and b

For your information the correct simplified equations are $a + b = 1$ and $-2a + b = -14$

Minimal evidence is required for this as it could be done on a GC. Just accept values of both a and b

A1 $a = 5$ and $b = -4$

Question Number	Scheme	Marks
3.(a)	$\frac{1}{3}x$ as the final answer.	B1
(b)	$81x^{-3}$ as the final answer	B1
(c)	$x^{\frac{3}{2}}$ as the final answer	B1
		[3] (3 marks)

In all parts of this question candidates do not have to explicitly state the values of k and n . Award the mark(s) as above.

If they go on to give incorrect values of k and n you may isw, but do not isw on incorrect index work Eg. $\frac{1}{3}x = x^{-3}$.

If candidates make two different attempts and give two (or more) different answers then please put these in review

(a)

B1 $\frac{1}{3}x$ but accept equivalent such as $\frac{x}{3}$, $\frac{1}{3} \times x$, $\frac{1}{3}x^1$ or $3^{-1}x$ etc.

(b)

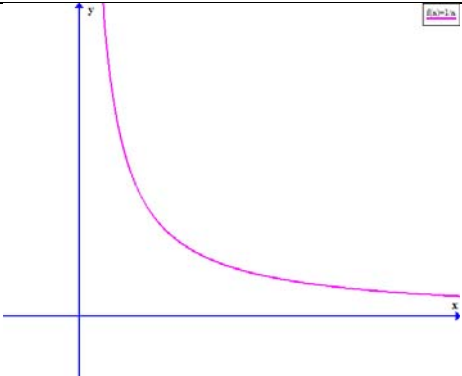
B1 $81x^{-3}$. Accept exact equivalents such as $81 \times x^{-3}$

Do not accept $\frac{81}{x^3}$ as the final answer unless it is preceded by $81x^{-3}$ as it is not in the form required by the question.

(c)

B1 $x^{\frac{3}{2}}$ but accept exact equivalents such as $1 \times x^{1.5}$

Do not accept $x x^{\frac{1}{2}}$ or $x\sqrt{x}$ as the final answer unless preceded by $x^{\frac{3}{2}}$ as they are not in the form required by the question.

Question Number	Scheme	Marks												
4 (a)	 <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>M1: General shape correct so part of rectangular hyperbola A1: Accuracy on asymptotes and no crossing of axes</p> </div>	<p>M1 A1</p> <p>[2]</p>												
(b)	<table border="1" style="margin-bottom: 10px;"> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">1.5</td> <td style="padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">2.5</td> <td style="padding: 2px 10px;">3</td> </tr> <tr> <td style="padding: 2px 10px;">y</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">0.667</td> <td style="padding: 2px 10px;">0.5</td> <td style="padding: 2px 10px;">0.4</td> <td style="padding: 2px 10px;">0.333</td> </tr> </table> <p>State $h = 0.5$, or use of $\frac{1}{2} \times 0.5$;</p> <p style="text-align: center;">$\{ 1 + 0.333 + 2(0.667 + 0.5 + 0.4) \}$</p> <p style="text-align: center;">$\frac{1}{2} \times 0.5 \times \{ 4.467 \} = \text{awrt } 1.12$</p> <p style="text-align: right;">For structure of $\{ \dots \}$;</p>	x	1	1.5	2	2.5	3	y	1	0.667	0.5	0.4	0.333	<p>B1 aef</p> <p>M1A1</p> <p>A1cao</p> <p>[4]</p> <p>(6 marks)</p>
x	1	1.5	2	2.5	3									
y	1	0.667	0.5	0.4	0.333									

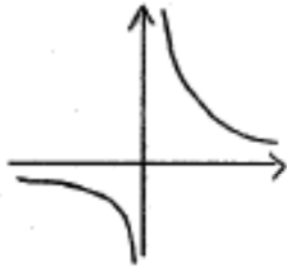
- (a)
- M1 General shape correct showing **part of a rectangular hyperbola in the first quadrant**. Condone for this mark the curve meeting or intersecting either axis. Condone incorrect asymptotes, for example at $y = 1$. See Practice, examples on the following page and Qualification for clarification
- A1 Do not allow for intersections with axes. Curve must appear to approach/ be asymptotic to both the x and y axes. Ignore sections where $x < 0$
- (b)
- B1 For using a strip width of 0.5. This may appear in a trapezium rule as $\frac{1}{2} \times 0.5$ or 0.25 or equivalent
- M1 Scored for the correct $\{ \dots \}$ bracket structure. It needs to contain the first y value +last y value with 2 times the sum of the remaining y values in the table (with no additional values). If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). It is M0 if values used in brackets are x values instead of y values
- A1 For the correct bracket $\{ \dots \}$. Accept $\{ 1 + 0.33 + 2(0.67 + 0.5 + 0.4) \}$ where candidate rounds to 2dp
- A1 For awrt 1.12.

NB: Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2} h(a + b)$ used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Special case: Bracketing mistake $0.25 \times (1 + 0.333) + 2(0.667 + 0.5 + 0.4)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given).

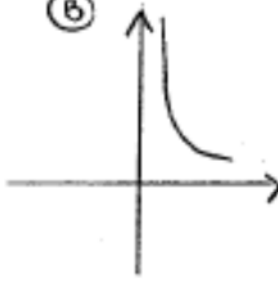
Q4 All score M1A1

(A)



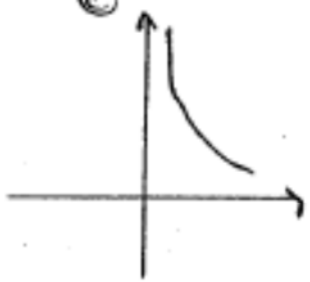
M1 shape ✓
A1 Accuracy ✓

(B)



M1 shape ✓
A1 condone lack of symmetry

(C)



M1 shape ✓
A1 - curvature going in the right direction

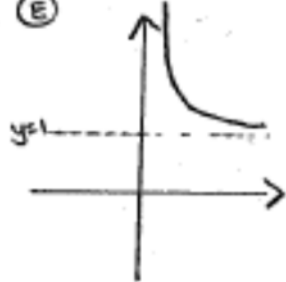
All score M1A0

(D)



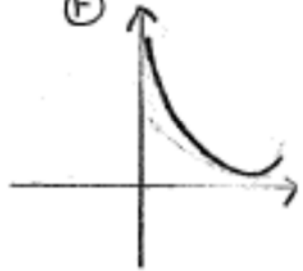
M1 part of hyperbola
A0 hits y axis and curvature incorrect at right hand end x

(E)



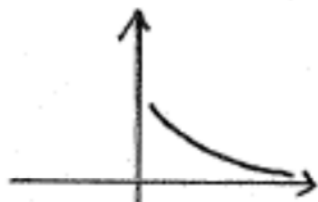
M1 shape ✓
A0 'too' far away from x axis (or asymptote not shown)

(F)



M1 - part of hyperbola
A0 - curvature at rhs incorrect.

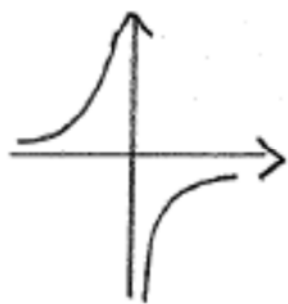
(G)



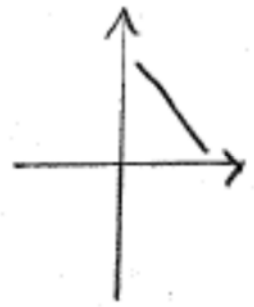
M1 - part of a hyperbola
A0 - curvature lacking accuracy at left hand end

Both score M0A0

(H)



(I)



Question Number	Scheme	Marks
5. (i)	Use or state $y \log 3 = \log 12$ or $\log_3 12$ Obtain $y = 2.26$ or awrt 2.26	M1 A1 [2]
(ii)	Use or state $\log_2 16 = 4$ or $2^4 = 16$ Use $\log_2(x+3) - \log_2(2x+4) = \log_2 \frac{(x+3)}{(2x+4)}$ Then $\frac{x+3}{(2x+4)} = 16$, and so $x = -\frac{61}{31}$	M1 A1, A1 [4]
		6 marks
Alt (ii)	$\log_2(x+3) - \log_2(2x+4) = \log_2 16$ $\log_2(x+3) = \log_2 16 + \log_2(2x+4)$ $\log_2(x+3) = \log_2 16(2x+4)$ $(x+3) = 16(2x+4)$ $x = -\frac{61}{31}$	B1 M1 A1 A1

(i)

M1 Uses power law for logs. This method may be implied by sight of $y \log 3 = \log 12$ or $\log_3 12$ The method may be scored for using trial and error provided you see $3^{2.26} = 11.9...$ and $3^{2.27} = 12.1...$

A1 awrt 2.26.

Just the answer with no (incorrect) working is 2 marks.

(ii)

B1 Connects 16 with '4' correctly. Evidence would be $\log_2 16 = 4$ or $2^4 = 16$ and could be scored at any time

M1 For the correct use of either the addition or subtraction law of logs

A1 Correct equation not involving logs

A1 cso $x = -\frac{61}{31}$

Special case:

Candidates who recover from "incorrect" log work and achieve the correct answer can score special case 1010.

$$\log_2(x+3) - \log_2(2x+4) = 4 \Rightarrow \frac{\log_2(x+3)}{\log_2(2x+4)} = \log_2 16 \Rightarrow \frac{x+3}{2x+4} = 16 \Rightarrow x = -\frac{61}{31}$$

Question Number	Scheme	Marks
6.(a)	$(2+ax)^6 = 2^6 + \binom{6}{1}2^5 \cdot (ax) + \binom{6}{2}2^4 \cdot (ax)^2 + \dots$ $= 64 + 192ax + 240a^2x^2 + \dots$	M1 B1, A1, A1 [4]
(b)	$192a = 240a^2$ $a = \frac{192}{240} = 0.8 \text{ or equivalent}$	M1 A1 [2]
Alt 6.(a)	$(2+ax)^6 = 2^6 \left(1 + \frac{a}{2}x\right)^6 = 2^6 \left(1 + 6 \times \frac{a}{2}x + \frac{6 \times 5}{2} \left(\frac{a}{2}x\right)^2 + \dots\right)$ $= 64 + 192ax + 240a^2x^2 + \dots$	M1 B1, A1, A1 [4]

- (a)**
- M1 The method mark is awarded for an attempt at a Binomial expansion to get an unsimplified second or third term – Look for a correct binomial coefficient multiplied by a correct power of x . Eg ${}^6C_1 \dots x$ or ${}^6C_2 \dots x^2$
Condone bracket errors or errors (or omissions) in the powers of 2.
Accept any notation for 6C_1 , 6C_2 , e.g. as on scheme or 6, and 15 from Pascal's triangle.
This mark may be given if no working is shown, if either or both of the terms including x is correct.
If the candidate attempts the expansion in descending powers allow ${}^6C_5 \dots x^5$ or ${}^6C_4 \dots x^4$ oe.
- In the alternative it is for the correct form inside the bracket accepting either $1 + 6 \times \frac{a}{2}x + \frac{6 \times 5}{2} \left(\frac{a}{2}x\right)^2$
or $1 + 6 \times \frac{a}{2}x + \frac{6 \times 5}{2} \frac{a}{2}x^2$
- B1 Must be simplified to 64 (writing just 2^6 is B0).
- A1 Score for either of $192ax$ or $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket
- A1 Score for both of $192ax$ and $240a^2x^2$ correct. Allow $240a^2x^2$ appearing as $240(ax)^2$ with the bracket
Allow listing of terms 64, $192ax$, $240a^2x^2$ for all 4 marks.
- (b)**
- M1 Score for setting the coefficients of their x and x^2 terms equal. They must reach an equation not involving x 's.
A1 This is also for any equivalent fraction or decimal to 0.8. Ignore any reference to $a=0$.

Question Number	Scheme	Marks
7. (a)	Length of the arc is 12 (cm)	B1 [1]
(b)	States or uses arc length '12' = $9 \times \theta$ $\theta = \frac{4}{3}$ or awrt 1.33	M1 A1 [2]
(c)	Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times \theta$ Area = awrt 54 (cm ²)	M1 A1
(d)	Area of triangle $OAB = \frac{1}{2} \times 9 \times 9 \times \sin \theta$, = awrt 39.4 cm ²	[2] M1, A1 [2] 7 marks

Mark parts (a) and (b) together. For example the arc length may be scored in part (b)

(a)

B1 12 (cm)

(b)

M1 Uses or states arc length $s = 9 \times \theta$ with their value of 's' from part (a)

Allow degree calculations for the M mark. Score for equivalents of $\frac{\theta}{12} = \frac{360}{2\pi \times 9}$ FYI $\theta \approx 76.4^\circ$

A1 For $\theta = \frac{4}{3}$ or equivalent such as $\theta = \frac{12}{9}, 1\frac{1}{3}$. Allow decimals awrt 1.33

Accept answers in terms of π but the accuracy must be either awrt 0.424π or 0.425π

(c)

M1 Uses or states area $A = \frac{1}{2}9^2 \times \theta$ with their value of ' θ ' in radians from part (b)

Allow for a correct formula in degrees. $A = \frac{\theta}{360} \times \pi 9^2$ with their value of ' θ ' in degrees from part (b)

A1 Area = awrt 54 (cm²)

(d)

M1 Correct method to find the area of triangle ABC , usually by using $\frac{1}{2} \times 9 \times 9 \times \sin \theta$ with their value of ' θ '

from part (b). Alternative methods are acceptable using the fact that triangle ABC is isosceles but it must be a full method involving the calculation of both base and perpendicular height by trigonometry twice or trigonometry once and Pythagoras once followed by $\frac{1}{2}bh$

The segment formula $\frac{1}{2} \times 9 \times 9 \times \theta - \frac{1}{2} \times 9 \times 9 \times \sin \theta$ will score M0 unless the candidate subtracts this from the area of a sector.

A1 Awrt = 39.4 cm²

Question Number	Scheme	Marks
8. (a)	$238 = a + kd$ or $108 = a + kd$ with any values for k	M1
	$238 = a + (14)d$ or $108 = a + (24)d$ or " d " = ± 13	A1
(b)	$238 = a + (14)d$ and $108 = a + (24)d$	A1
	Solves their simultaneous equations to obtain $a =$; (so a) = 420	M1; A1 [5]
	Uses $\frac{25}{2}(2 \times a + (25-1) \times "-13")$ or Uses $\frac{25}{2}(a + 108)$, to obtain = 6600	M1, A1 [2]
		7 marks

(a)
M1 Score for $238 = a + kd$ or $108 = a + kd$ with any non- zero integer value for k

A1 One of $238 = a + (14)d$, or $108 = a + (24)d$ or " d " = ± 13
The " d " = ± 13 can be achieved from equations such as $238 = a + (13)d$, or $108 = a + (23)d$

A1 Both $238 = a + (14)d$, and $108 = a + (24)d$

M1 Finds the value of a by solving a pair of simultaneous equations in a and d

A1 Achieves ($a =$) 420

In an alternative, working by a method of differences, you may see very few formulae: The scheme can be easily applied,

1st M1 Seeing $\frac{238-108}{10}$ or $\frac{108-238}{10}$

1st A1 For ± 13

2nd A1 For $238 + 13 \times 14$ or $108 + 13 \times 24$.

Note that this is achieved after the award of the next mark. It is scored as the third mark of (a) on e –pen.

2nd M1 Sight of $238 + 'k' \times d$ or $108 + 'k' \times d$ with any non- zero integer values for k and their d

3rd A1 Achieves 420

(b)

M1 Uses a correct sum formula $S = \frac{n}{2}(2a + (n-1)d)$ with $n = 25$ and their values of a and d

Alternatively uses $S = \frac{n}{2}(a + l)$ with $n = 25, l = 108$ and their value of a .

A1 cao 6600

Question Number	Scheme	Marks
9(a)	<p>Uses $b^2 - 4ac = (6k+4)^2 - 4 \times 1 \times 3$</p> <p>Multiplies out and uses $b^2 - 4ac < 0$</p> <p style="padding-left: 40px;">to give $36k^2 + 48k + 16 - 12 < 0$ so $9k^2 + 12k + 1 < 0$ *</p> <p>(b) Solves quadratic by formula or completion of square to give $k =$</p> <p>Obtains $k = \frac{-12 \pm \sqrt{108}}{18}$ or accept $k = \text{awrt } -1.24, -0.09$</p> <p>Chooses region between two values and deduces $\frac{-2 - \sqrt{3}}{3} < k < \frac{-2 + \sqrt{3}}{3}$</p>	<p>B1</p> <p>M1</p> <p>A1 *</p> <p style="text-align: right;">[3]</p> <p>M1</p> <p>A1</p> <p>M1 A1 cao</p> <p style="text-align: right;">[4]</p> <p>7 marks</p>

(a)

B1 Uses $b^2 - 4ac = (6k + 4)^2 - 4 \times 1 \times 3$ or equivalentM1 Attempt to multiply out and use their $b^2 - 4ac < 0$ or $b^2 < 4ac$ Alternatively they can find $b^2 - 4ac$ and then state $b^2 - 4ac < 0$ or $b^2 < 4ac$ A1* Achieves $9k^2 + 12k + 1 < 0$ This is a given answer and you must check that all aspects are correct. **In all cases you should expect to see an intermediate line(s) (including < 0) before the final answer.** $b^2 - 4ac = 4(9k^2 + 12k + 1)$. As $b^2 - 4ac < 0 \Rightarrow 9k^2 + 12k + 1 < 0$ is fine for B1 M1 A1* $b^2 - 4ac = 4(9k^2 + 12k + 1) \Rightarrow 9k^2 + 12k + 1 < 0$ is B1 M0 A0*

(b)

M1 Solves the equation $9k^2 + 12k + 1 = 0$ by formula or completing the square. Factorisation is not a suitable method in this case and scores M0. The answers could just appear from a graphical calculator.Sight of either root, $\frac{-12 - \sqrt{108}}{18}$ or $\frac{-12 + \sqrt{108}}{18}$ is evidence that the formula has been used.Sight of $-\frac{6}{9} + \sqrt{\frac{27}{81}}$ or could be evidence that completion of square has been used.A1 Accept $k = \frac{-12 - \sqrt{108}}{18}$ and $\frac{-12 + \sqrt{108}}{18}$ or decimal equivalents awrt 2dp -1.24 and -0.09

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation.

This can be awarded for an inside region appearing in the form $\frac{-2 - \sqrt{3}}{3} \leq k \leq \frac{-2 + \sqrt{3}}{3}$ A1 cao $\frac{-2 - \sqrt{3}}{3} < k < \frac{-2 + \sqrt{3}}{3}$. Accept $-\frac{2}{3} - \frac{1}{\sqrt{3}} < k < -\frac{2}{3} + \frac{1}{\sqrt{3}}$ Accept equivalents such as $\left(\frac{-2 - \sqrt{3}}{3}, \frac{-2 + \sqrt{3}}{3}\right)$ $k > \frac{-2 - \sqrt{3}}{3}$ and $k < \frac{-2 + \sqrt{3}}{3}$ even $\left\{\frac{-2 - \sqrt{3}}{3} \cap \frac{-2 + \sqrt{3}}{3}\right\}$ Do not accept $\left[\frac{-2 - \sqrt{3}}{3}, \frac{-2 + \sqrt{3}}{3}\right]$ $k > \frac{-2 - \sqrt{3}}{3}$ or $k < \frac{-2 + \sqrt{3}}{3}$ and $k > \frac{-2 - \sqrt{3}}{3}, k < \frac{-2 + \sqrt{3}}{3}$

including the version of the final one without the comma

If the candidate writes $\frac{-2 - \sqrt{3}}{3} < x < \frac{-2 + \sqrt{3}}{3}$ this can be awarded M1 A0

Question Number	Scheme	Marks
10 (a)	$u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$	M1, A1 [2]
(b)	$u_{20} = 4 \times \left(\frac{2}{3}\right)^{19}$; = 0.00180 or 0.0018 or exact equivalent	M1; cao A1 [2]
(c)	Use $\sum_{i=1}^{16} u_i = \frac{4(1 - (\frac{2}{3})^{16})}{1 - \frac{2}{3}}$ Find 12 - <i>their</i> $\sum_{i=1}^{16} u_i$ = 12 - 11.9817 = awrt 0.0183	M1 dM1 A1 [3]
(d)	12 is the sum to infinity (and all terms are positive) so sum is less than 12 Or $\sum_{i=1}^n u_i = \frac{4(1 - (\frac{2}{3})^n)}{1 - \frac{2}{3}} = 12 - 12(\frac{2}{3})^n$ and $(\frac{2}{3})^n > 0$ so is less than 12	B1 [1]
		[8 marks]

(a)
M1 Any one term is 2/3 the previous term. Accept for example $u_2 = \text{awrt } 2.67$

A1 All 3 terms correct. Accept exact equivalents $u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$

(b)
M1 Uses correct nth term formula ar^{n-1} with $a = 4$, $n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Condone for the M mark use of ar^{n-1} with $a = \frac{8}{3}$ (awrt 2.67), $n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Expressions such as $4 \times (\frac{2}{3})^{19}$, $\frac{8}{3} \times (\frac{2}{3})^{18}$ and $\frac{2^{n+1}}{3^{n-1}} \rightarrow \frac{2^{21}}{3^{19}}$ are correct and sufficient for M1

A1 Accept any of 0.0018, 0.00180, 1.80×10^{-3} or 1.8×10^{-3}

(c)
M1 Uses the correct sum formula $S = \frac{a(r^n - 1)}{(r - 1)}$ or $S = \frac{a(1 - r^n)}{(1 - r)}$ with $a = 4$, $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

Condone the sum formula $S = \frac{a(r^n - 1)}{(r - 1)}$ or $S = \frac{a(1 - r^n)}{(1 - r)}$ with $a = \frac{8}{3}$ (awrt 2.67), $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

dM1 Dependent upon the previous M mark. Score for an attempt at finding $12 - \sum_{i=1}^{16} u_i$

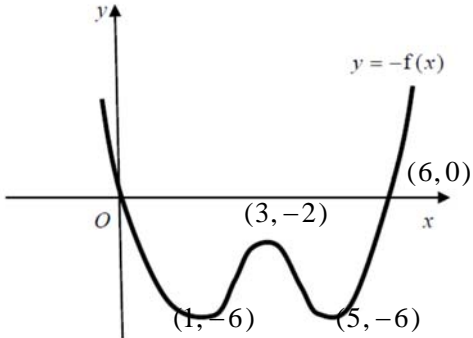
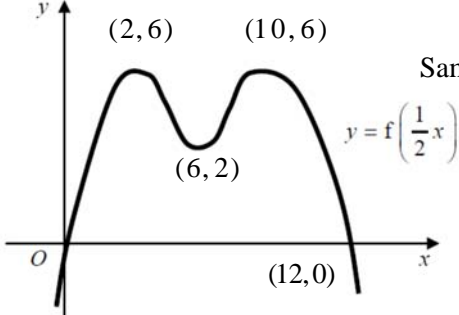
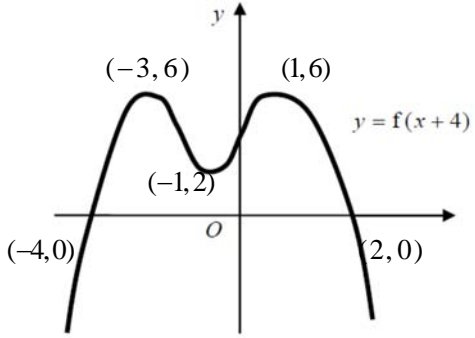
A1 awrt 0.0183

Note: Some candidates may list all 16 terms which is acceptable provided the answer is accurate

(d)
B1 Need a reason + a minimal conclusion. Eg The sum to infinity = 12 **and** sum is less than 12
Allow sum to infinity is 12, hence true.

Question Number	Scheme	Marks
11.	$f'(x) = 3\sqrt{x} - \frac{9}{\sqrt{x}} + 2$	
(a)	$3\sqrt{x} - \frac{9}{\sqrt{x}} + 2 = 3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$ $f(x) = \int (3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2) dx = \left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x \right) (+c)$ <p>When $x = 9$, $y = 14$ so $c =$ $\Rightarrow f(x) = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$</p>	B1 M1 A1 A1 M1 A1 [6]
(b)	<p>Gradient of curve at P is $f'(9) = 8$ Gradient of normal is $k = -\frac{1}{f'(9)}$</p> <p>Equation of normal is $(y - 14) = k(x - 9)$ or $y = kx + c'$ with use of $(9, 14)$ to find c' This is $(y - 14) = -\frac{1}{8}(x - 9)$ or $y = -\frac{1}{8}x + 15\frac{1}{8}$ So $x + 8y - 121 = 0$</p>	B1 M1 dM1 A1 A1 [5] 11 marks

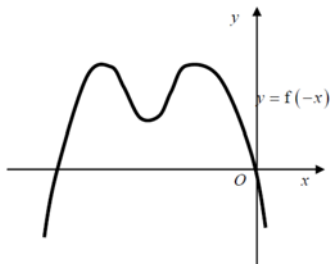
- (a)
- B1 Expression written as $3x^{\frac{1}{2}} - 9x^{-\frac{1}{2}} + 2$ (May be implied by correct integration)
- M1 Evidence of integration, so $x^n \rightarrow x^{n+1}$ at least once. Accept on the 2 and follow through on incorrect powers.
- A1 Two terms of $\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct. There is no need to simplify or have $+c$
- A1 All three terms of $c \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} + 2x$ correct with no need to simplify or have $+c$.
- M1 Puts $x = 9$ and $y = 14$ into a changed expression (with $+c$) to obtain $c =$
- A1 Needs each term simplified and c correct. Allow simplified equivalents of $f(x) = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$
 For example allow $y = 2x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + 2x - 4$ and $y = 2x\sqrt{x} - 18\sqrt{x} + 2x - 4$.
 Remember to isw after a correct answer
- (b)
- B1 $f'(9) = 8$
- M1 Uses or states that the gradient of the normal is $-\frac{1}{f'(9)}$
- dM1 Finds the equation of a line using $(9, 14)$ and their $-\frac{1}{f'(9)}$ as the numerical gradient.
 It is dependent upon the previous M
 Allow for $(y - 14) = k(x - 9)$ or $y = kx + c'$ where $k = -\frac{1}{f'(9)}$
- A1 A correct unsimplified answer such as $(y - 14) = -\frac{1}{8}(x - 9)$ or $y = -\frac{1}{8}x + \frac{121}{8}$
- A1 For any multiple of $x + 8y - 121 = 0$

Question Number	Scheme	Marks
<p>12.(a)</p>	 <p style="text-align: right;">$y = -f(x)$</p>	<p>W shape B1</p> <p>Crosses the x axis at $(0,0)$ and $(6,0)$ B1</p> <p>All 3 turning points correct B1</p> <p style="text-align: right;">[3]</p>
<p>(b)</p>	 <p style="text-align: right;">$y = f\left(\frac{1}{2}x\right)$</p>	<p>Same shape passing through (or starting at) the origin B1</p> <p>Cuts the x axis at $(12,0)$ B1</p> <p>All 3 turning points correct B1</p> <p style="text-align: right;">[3]</p>
<p>(c)</p>	 <p style="text-align: right;">$y = f(x+4)$</p>	<p>Translation left or right B1</p> <p>Cuts the x axis at $(2,0)$ and $(-4,0)$ B1</p> <p>All 3 turning points in correct position with correct coordinates B1</p> <p style="text-align: right;">[3]</p>
		9 marks

Note: There are candidates who sketch graphs in which the curve just sits on (and does not pass through) the x -axis. Applying the scheme would mean that such a candidate would not gain the second mark in each case. We are ruling that you are only to withhold the mark **the first time** it occurs, so potentially a candidate could score 8 out of 9.

- (a)
- B1 For a W shape
- B1 For a curve **crossing** the x -axis at $(0, 0)$ and $(6, 0)$ **only**. Accept if the curve passes through the origin and $(6, 0)$ with only '6' marked. Condone $(0, 6)$ being marked on x axis. Accept if the coordinates of the points are given in the body of the script as say $P = (6, 0)$ and P being marked in the correct place on the diagram. Do not award if the curve meets the x axis at an additional point.
- B1 All three turning points correctly marked; maximum = $(3, -2)$ and minima $(1, -6)$ and $(5, -6)$. Do not allow the coordinates being transposed.

Special case: If the candidate sketches $y = f(-x)$ correctly with all coordinates correct they can score B1B0B0



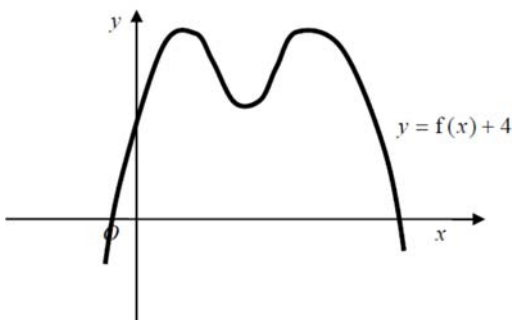
Maximum points = $(-1, 6)$ and $(-5, 6)$
Crosses x -axis at the origin and $(-6, 0)$
 Minimum point = $(-3, 2)$

- (b)
- B1 Score for the same 'shape' passing through/ or starting at the origin. The two maxima and one minimum must be in Quadrant 1
- B1 Curve **crosses** the x -axis at the origin and $(12, 0)$ only. See part (a) for allowable alternatives.
- B1 All three turning points correctly marked; minimum = $(6, 2)$ and maxima $(2, 6)$ and $(10, 6)$. Do not allow the coordinates being transposed.

Note: There is no special case as a candidate sketching $y = \frac{1}{2}f(x)$ will score B1B0B0 under the scheme anyway.

- (c)
- B1 For an attempted translation of the whole curve left or right. Award if the y coordinates of the turning points have stayed at 6 and 2 at least one x coordinate has changed. The curve does not need to meet or cross the x axis.
- B1 Curve **crosses** the x -axis at $(2, 0)$ and $(-4, 0)$ only
- B1 All 3 turning points correct and curve positioned correctly; Maximum point in quadrant one at $(1, 6)$
 Maximum point in quadrant two at $(-3, 6)$ and minimum point in quadrant two at $(-1, 2)$

Special case for candidate sketching $y = f(x) + 4$ with all coordinates correct can score B1B0B0



Maximum points = $(1, 10)$ and $(5, 10)$

Crosses x -axis either side of the origin with neither being given values.

Minimum point = $(3, 6)$

Question Number	Scheme	Marks
13. (i)	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x$ $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - (1 - \cos^2 x) \cos x - (1 - \sin^2 x) \sin x$ $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$ *	1 st M1 2 nd M1 A1 *
Alt I (i)	Use LHS = $(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$ $\equiv \sin^3 x + \sin x \cos^2 x - \sin^2 x \cos x + \sin^2 x \cos x - \cos^2 x \sin x + \cos^3 x$ $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	[3] 2 nd M1 1 st M1 A1 * [3]
Alt II (i)	Use RHS $\equiv \sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$ $= (\sin x + \cos x)(1 - \sin x \cos x)$	M1 M1 A1 [3]
(ii)	Use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ to give $3 \sin \theta = \frac{\sin \theta}{\cos \theta}$ $\cos \theta = \frac{1}{3}$ and $\sin \theta = 0$ So $\theta = 70.5, 289.5, \text{ or } 0 \text{ and } 180$ (do not require degrees symbol)	M1 A1 A1 M1 A1, <u>B1</u> [6] 9 marks

(i)

M1 Expands bracket to form 4 terms - condone sign slips but terms must be correct.

Allow $\sin x \sin x \cos x$ for $\sin^2 x \cos x$ and condone $\cos x^2$ for $\cos^2 x$ M1 Replaces $\sin^2 x$ by $(1 - \cos^2 x)$ and $\cos^2 x$ by $(1 - \sin^2 x)$ Condone $\cos x^2$ for $\cos^2 x$. This mark could be seen preceding the line $\sin x(1 - \cos^2 x) + \cos x(1 - \sin^2 x)$.A1* Completes proof with no errors. This is a given answer. Withhold this mark if poor notation or mixed variables are seen. Examples would be $\sin x + \cos x - (1 - \cos^2) \cos x - (1 - \sin^2) \sin x$ Don't accept for the A1* $\cos x^2$ for $\cos^2 x$ unless it is clearly bracketed.

(ii)

M1 Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or equivalent to give $3 \sin \theta = \frac{\sin \theta}{\cos \theta}$ or $3 \cos \theta = 1$ A1 Achieves $\cos \theta = \frac{1}{3}$ A1 Achieves $\sin \theta = 0$ M1 For $\arccos\left(\text{their } \frac{1}{3}\right)$ leading to one value of θ to the nearest degree. (You may need a calculator to check this)A1 For $\theta = \text{awrt } 70.5^\circ, 289.5^\circ$. This mark is withheld for extra solutions of $\arccos\left(\frac{1}{3}\right)$ in the range.

Ignore extra solutions outside the range.

B1 $\theta = 0, 180^\circ$ This mark is withheld for extra solutions arising from $\sin \theta = 0$ in the range.

Ignore extra solutions outside the range.

Note: Students who proceed from $\frac{\sin \theta}{\tan \theta} = \cos \theta$ can score M1A1A0M1A1B0 for 4 out of 6

Radian solutions, withhold only the final A1 mark. For your information solutions are awrt 2dp 1.23, 5.05, 0, 3.14 = (pi)

Question Number	Scheme	Marks		
14. (a)	$4x + 3 = 2x^{\frac{3}{2}} - 2x + 3$ so $6x = 2x^{\frac{3}{2}}$ $x = 9$ Points are (0, 3) and (9, 39)	M1 A1 B1, B1 [4]		
(b)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> <p>Method 1</p> $\int 2x^{\frac{3}{2}} - 2x + 3dx = \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Area of trapezium = $\frac{1}{2} \times (3 + 39) \times 9$</p> <p style="text-align: center;">OR $2 \times 9^2 + 3 \times 9$</p> <p>Use = Area of trapezium – Area beneath curve</p> <p>= 189 – 140.4 = 48.6</p> </td> <td style="width: 50%; vertical-align: top; border-left: 1px solid black;"> <p>Method 2</p> $\pm \left(\int 6x - 2x^{\frac{3}{2}} dx = 3x^2 - \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} \right)$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Implied by final answer of 48.6 or -48.6</p> <p>Implied by subtraction in the integration</p> <p>= 48.6</p> </td> </tr> </table>	<p>Method 1</p> $\int 2x^{\frac{3}{2}} - 2x + 3dx = \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Area of trapezium = $\frac{1}{2} \times (3 + 39) \times 9$</p> <p style="text-align: center;">OR $2 \times 9^2 + 3 \times 9$</p> <p>Use = Area of trapezium – Area beneath curve</p> <p>= 189 – 140.4 = 48.6</p>	<p>Method 2</p> $\pm \left(\int 6x - 2x^{\frac{3}{2}} dx = 3x^2 - \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} \right)$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Implied by final answer of 48.6 or -48.6</p> <p>Implied by subtraction in the integration</p> <p>= 48.6</p>	M1A1 M1 B1 M1 A1cso [6] (10 marks)
<p>Method 1</p> $\int 2x^{\frac{3}{2}} - 2x + 3dx = \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Area of trapezium = $\frac{1}{2} \times (3 + 39) \times 9$</p> <p style="text-align: center;">OR $2 \times 9^2 + 3 \times 9$</p> <p>Use = Area of trapezium – Area beneath curve</p> <p>= 189 – 140.4 = 48.6</p>	<p>Method 2</p> $\pm \left(\int 6x - 2x^{\frac{3}{2}} dx = 3x^2 - \frac{2}{\frac{5}{2}}x^{\frac{5}{2}} \right)$ <p>Uses their upper limit (and subtracts lower limit, often 0) to obtain an area</p> <p>Implied by final answer of 48.6 or -48.6</p> <p>Implied by subtraction in the integration</p> <p>= 48.6</p>			

(a)

M1 Set $4x + 3 = 2x^{\frac{3}{2}} - 2x + 3$, collects terms and forms an equation equivalent to $Ax = Bx^{\frac{3}{2}}$, $A, B \neq 0$

A1 Obtains $x = 9$ from a correct equation. You may ignore any reference to $x = 0$

B1 One of (0, 3), (9, 39)

B1 Both (0, 3) and (9, 39)

Note: This question requires the use of algebra to find (0, 3), (9, 39). Just the answers with no equation scores 0,0,1,1.

Similarly $2x^{\frac{3}{2}} - 2x + 3 = 4x + 3$ must be simplified before M1A1 can be scored.

Once $2x^{\frac{3}{2}} = 6x$ has been reached you can accept $x = 9$ without any real working as it could be done in your head.

(b)

M1 Attempted integration on either $\int 2x^{\frac{3}{2}} - 2x + 3dx$ or $\int (4x + 3) - (2x^{\frac{3}{2}} - 2x + 3)dx$ either way around

Accept as evidence $x^n \rightarrow x^{n+1}$ on any term

A1 Correct integration-may be unsimplified.

Accept $\frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x$ or $\left(4\frac{x^2}{2} + 3x \right) - \left(\frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - x^2 + 3x \right)$ either way around.

M1 Uses their upper limit (and subtracts the lower limit or vice versa) in their integrated function. If the lower limit is 0 you do not have to see the subtraction.

B1 Area of trapezium = 189 or $\frac{1}{2} \times (3 + 39) \times 9$ or $2 \times 9^2 + 3 \times 9$ from $\int 4x + 3dx$

This may be implied by the correct answer of 48.6 or (-48.6) in the alternative method.

M1 Uses area of trapezium - area under curve (either way around). It is usually implied by line 1 in the alternative method. The mark is scored for a correct method of finding the shaded area,

A1 48.6.

Note -48.6 is A0 even if candidate loses the sign

Question number	Scheme	Marks
15 (a)	$\sqrt{(12-5)^2 + (7-6)^2} = \sqrt{50} \text{ or } 5\sqrt{2}$	M1, A1 [2]
(b)	See $(x \pm 5)^2 + (y \pm 6)^2 = (\text{their numerical } r)^2$ $(x - 5)^2 + (y - 6)^2 = 50$	M1 B1, A1 [3]
(c)	Gradient of $AP = \frac{1}{7}$ So gradient of tangent is -7 Equation of tangent is $(y - 7) = -7(x - 12)$	B1 M1 dM1 A1 [4]
(d)	$AB = \sqrt{180} = (6\sqrt{5}), BC = \sqrt{160} = (4\sqrt{10}), AC = 10$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{160 + 100 - 180}{20\sqrt{160}}$ So $C = \text{awrt } 71.6$	M1 A1 A1 M1 A1 [5] 14 marks

(a)

M1 Sight of $\sqrt{(12-5)^2 + (7-6)^2}$ or $r^2 = 7^2 + 1^2$ or equivalent. It may be implied by $\sqrt{50}$ or $5\sqrt{2}$ or awrt 7.1

A1 $\sqrt{50}$ or $5\sqrt{2}$

(b)

M1 For the form $(x \pm 5)^2 + (y \pm 6)^2 = (\text{their numerical } r)^2$

Accept a form $x^2 + y^2 \pm 10x \pm 12y + c = 0$ where $c = 61 - (\text{their numerical } r^2)$

B1 For $(x-5)^2 + (y-6)^2 = ..$ or $x^2 + y^2 - 10x - 12y + ... = 0$ oe

This can be awarded from $(x-5)^2 + (y-6)^2 = r^2$ with an algebraic 'r'

A1 Completely correct answer $(x-5)^2 + (y-6)^2 = 50$

$(x-5)^2 + (y-6)^2 = (5\sqrt{2})^2$ and $x^2 + y^2 - 10x - 12y + 11 = 0$ are acceptable alternatives.

(c)

B1 Gradient $AP = \frac{1}{7}$

M1 For using the negative reciprocal of the gradient for AP in finding the gradient of the tangent

dM1 Dependent upon the previous M1. It is for a linear equation through $(12,7)$ with their negative reciprocal gradient. If the form $y = mx + c$ it must be a full method to find 'c'

A1 Accept any unsimplified form and remember to isw. Accept $y - 7 = -7(x - 12)$, $y = -7x + 91$,

(d)

M1 A correct attempt to find the length of **any** line using the 'difference' between the coordinates.

Accept any of $AB = \sqrt{12^2 + 6^2} (\sqrt{180})$, $BC = \sqrt{12^2 + 4^2} (\sqrt{160})$, $AC = \sqrt{8^2 + 6^2} (\sqrt{100} = 10)$

This would be implied by the sight of. $AB = \text{awrt } 13.4$, $BC = \text{awrt } 12.6$, $AC = 10$

A1 Two lengths 'correct' either exact or awrt 3 sf $AB = \sqrt{180} = 13.4$, $BC = \sqrt{160} = 12.6$, $AC = 10$

A1 All three lengths 'correct' either exact or awrt 3 sf $AB = \sqrt{180} = 13.4$, $BC = \sqrt{160} = 12.6$, $AC = 10$

M1 Correct application of the cosine rule with their AB, AC, BC . Look for $\cos C = \frac{BC^2 + AC^2 - AB^2}{2 \times BC \times AC}$ with their lengths in the correct positions.

Allow the method of finding $\cos C$ from $AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \cos C$ with their lengths in the correct positions. You may condone rearrangement errors in this method.

A1 Accept awrt $C = 71.6$

Question Number	Scheme	Marks
16. (a)	$\pi R^2 H + \frac{2}{3} \pi R^3 = 800\pi$ so $H = \frac{800}{R^2} - \frac{2}{3} R$ *	M1 A1*
(b)	$A = \pi R^2 + 2\pi RH + 2\pi R^2$ $A = 3\pi R^2 + 2\pi R \left(\frac{800}{R^2} - \frac{2}{3} R \right)$ so $A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ *	B1 M1 A1 *
(c)	Find $\frac{dA}{dR} = \frac{10}{3} \pi R - \frac{1600\pi}{R^2}$ Put derivative equal to zero and obtain $R^3 = 480$ So $R = 7.83$	[5] M1 A1 dM1 A1 A1
(d)	Consider $\frac{d^2 A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3} > 0$ so minimum	[5] M1A1
(e)	$H = \text{awrt } 7.83$	[2] B1
		[1]
		13 marks

(a)
 M1 Sets up volume equation with $800\pi = \pi R^2 H + \frac{2}{3} \pi R^3$ and attempts to make H the subject. Condone 800 instead of 800π . Accept for this mark lower case letters $800\pi = \pi r^2 H + \frac{2}{3} \pi r^3$ and a lack of consistency in lettering.

A1* This is a show that question and there must be an intermediate line showing (or implying) a division of $\pi r^2 / \pi R^2$. Lettering must be correct and consistent from the point where you see $800\pi = \dots$.
 Examples of an intermediate line are;

$$800\pi = \pi R^2 H + \frac{2}{3} \pi R^3 \Rightarrow H = \frac{800\pi - \frac{2}{3} \pi R^3}{\pi R^2} \Rightarrow H = \frac{800}{R^2} - \frac{2}{3} R$$

$$800\pi = \pi R^2 H + \frac{2}{3} \pi R^3 \Rightarrow \frac{800}{R^2} = H + \frac{2}{3} R \Rightarrow H = \frac{800}{R^2} - \frac{2}{3} R$$

(b)
 B1 A correct expression for the surface area containing three separate correct elements

Allow either $A = \pi R^2 + 2\pi RH + 2\pi R^2$ or $A = \pi R^2 + 2\pi RH + \frac{4\pi R^2}{2}$

Allow lower case lettering for this mark

M1 Score for replacing $H = \frac{800}{R^2} - \frac{2}{3} R$ in their expression for A which must be of the form,

$$A = B\pi R^2 + C\pi RH, \quad B, C \in \mathbb{N}, \text{ condoning missing brackets.}$$

A1* This is a show that question and all aspects must be correct. Lettering in (b) must be consistent and correct from the point at which $\frac{800}{R^2} - \frac{2}{3} R$ is substituted. Do not, however, withhold a second mark for using lower case letters if it has been withheld in part (a) for mixed lettering.

Accept $A = 2\pi R^2 + \pi R^2 + 2\pi R \left(\frac{800}{R^2} - \frac{2}{3} R \right) \Rightarrow A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ with little or no evidence

(c)

M1 Evidence of differentiation so sight of $R^2 \rightarrow R$ or $R^{-1} \rightarrow R^{-2}$

A1 Both terms correct. Does not need to be simplified. $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$

Do not be concerned by inconsistent lettering and condone incorrect notation on the lhs such as $\frac{dy}{dx} =$

dM1 Sets $\frac{dA}{dR} = 0$ and proceeds to $R^n = C$ This is dependent upon the previous M

A1 A correct intermediate answer. $R^3 = 480$. This may be implied by a correct answer following a correct derivative.

A1 Allow awrt $R = 7.83$

(d)

M1 Achieves a correct second derivative $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$

Or attempts to substitute their positive R in their second derivative (usual rules for differentiation having been applied to at least one term)

Or alternatively attempts to find the numerical value of the gradient either side of their value of R

Or alternatively attempts to find the numerical value of V at R and either side of R

A1 This requires (1) a correct function, (2) a correct statement and (3) a correct conclusion

Either (1) $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$ (2) At their $R = + \dots$, $\frac{d^2A}{dR^2} = + \dots > 0$ (3) hence minimum.

Or (1) $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3}$, (2) as $R > 0$, $\frac{d^2A}{dR^2} > 0$ (3) hence minimum

Or (1) $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$ (2) at $R = 7$ $\frac{dA}{dR} = -29 < 0$ at $R = 8$ $\frac{dA}{dR} = 5 > 0$ (3) Hence minimum

The table below can be used to check most calculations

Value of R	7	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
Gradient at R	-29	-25	-22	-18	-14	-11	-7	-4	-1	2	5

Or (1) $A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ (2), $A_{7.82} > A_{7.83} < A_{7.84}$ with evidence (see below), (3) Hence minimum

Value of R	7.8	7.81	7.82	7.83	7.84	7.85	7.86	7.87	7.88	7.89	7.9
Value of A	962.50	962.49	962.49	962.48	962.49	962.49	962.50	962.51	962.52	962.54	962.56

(e)

B1 $H = \text{awrt } 7.83$

