



Oxford Cambridge and RSA

**Wednesday 5 June 2019 – Morning**

**A Level Mathematics A**

**H240/01 Pure Mathematics**

**Time allowed: 2 hours**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Solve the inequality  $10x^2 + x - 2 > 0$ . [4]

**2** The point  $A$  is such that the magnitude of  $\overrightarrow{OA}$  is 8 and the direction of  $\overrightarrow{OA}$  is  $240^\circ$ .

(a) (i) Show the point  $A$  on the axes provided in the Printed Answer Booklet. [1]

(ii) Find the position vector of point  $A$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [3]

The point  $B$  has position vector  $6\mathbf{i}$ .

(b) Find the exact area of triangle  $AOB$ . [2]

The point  $C$  is such that  $OABC$  is a parallelogram.

(c) Find the position vector of  $C$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [2]

**3** The function  $f$  is defined by  $f(x) = (x - 3)^2 - 17$  for  $x \geq k$ , where  $k$  is a constant.

(a) Given that  $f^{-1}(x)$  exists, state the least possible value of  $k$ . [1]

(b) Evaluate  $ff(5)$ . [2]

(c) Solve the equation  $f(x) = x$ . [3]

(d) Explain why your solution to part (c) is also the solution to the equation  $f(x) = f^{-1}(x)$ . [1]

**4** Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.

(a) Find Sam's salary in the tenth year. [2]

(b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. [4]

(c) Comment on how realistic this model may be in the long term. [1]

- 5 A curve has equation  $x^3 - 3x^2y + y^2 + 1 = 0$ .
- (a) Show that  $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$ . [4]
- (b) Find the equation of the normal to the curve at the point (1, 2). [4]

- 6 Let  $f(x) = 2x^3 + 3x$ . Use differentiation from first principles to show that  $f'(x) = 6x^2 + 3$ . [6]

7 **In this question you must show detailed reasoning.**

A sequence  $u_1, u_2, u_3 \dots$  is defined by  $u_n = 25 \times 0.6^n$ .

Use an algebraic method to find the smallest value of  $N$  such that  $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^N u_n < 10^{-4}$ . [8]

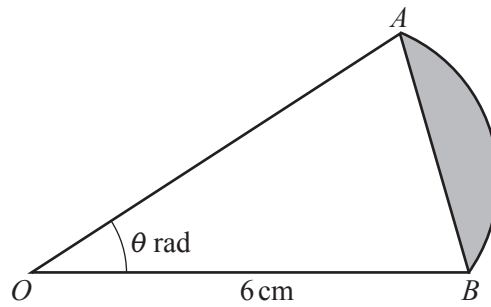
- 8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is  $x$  m at time  $t$  seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When  $t = 100$ ,  $x = 0.64$  and, at this instant, the height is decreasing at a rate of  $0.0032 \text{ ms}^{-1}$ .

- (a) Show that  $\frac{dx}{dt} = -0.004\sqrt{x}$ . [2]
- (b) Find an expression for  $x$  in terms of  $t$ . [4]
- (c) Hence determine at what time, according to this model, the tank will be empty. [2]
- 9 (a) Express  $3 \cos 3x + 7 \sin 3x$  in the form  $R \cos(3x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]
- (b) Give full details of a sequence of three transformations needed to transform the curve  $y = \cos x$  to the curve  $y = 3 \cos 3x + 7 \sin 3x$ . [4]
- (c) Determine the **greatest** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]
- (d) Determine the **least** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]

10



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius  $6$  cm.

The angle  $AOB$  is  $\theta$  radians.

The area of the segment bounded by the chord  $AB$  and the arc  $AB$  is  $7.2$  cm<sup>2</sup>.

(a) Show that  $\theta = 0.4 + \sin \theta$ . [3]

(b) Let  $F(\theta) = 0.4 + \sin \theta$ .

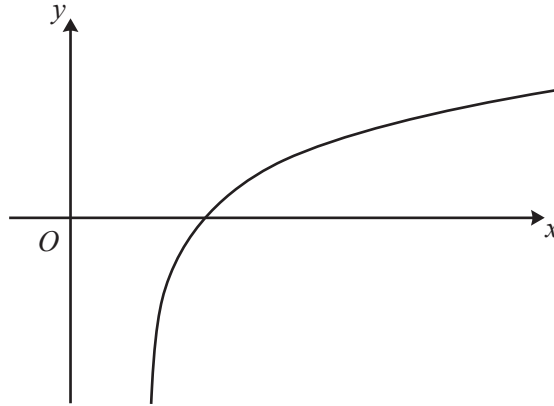
By considering the value of  $F'(\theta)$  where  $\theta = 1.2$ , explain why using an iterative method based on the equation in part (a) will converge to the root, assuming that  $1.2$  is sufficiently close to the root. [2]

(c) Use the iterative formula  $\theta_{n+1} = 0.4 + \sin \theta_n$  with a starting value of  $1.2$  to find the value of  $\theta$  correct to 4 significant figures.

You should show the result of each iteration. [3]

(d) Use a change of sign method to show that the value of  $\theta$  found in part (c) is correct to 4 significant figures. [3]

11



The diagram shows part of the curve  $y = \ln(x-4)$ .

- (a) Use integration by parts to show that  $\int \ln(x-4) dx = (x-4)\ln|x-4| - x + c$ . [5]
- (b) State the equation of the vertical asymptote to the curve  $y = \ln(x-4)$ . [1]
- (c) Find the total area enclosed by the curve  $y = \ln(x-4)$ , the  $x$ -axis and the lines  $x = 4.5$  and  $x = 7$ . Give your answer in the form  $a \ln 3 + b \ln 2 + c$  where  $a$ ,  $b$  and  $c$  are constants to be found. [4]

12 A curve has equation  $y = a^{3x^2}$ , where  $a$  is a constant greater than 1.

- (a) Show that  $\frac{dy}{dx} = 6xa^{3x^2} \ln a$ . [3]
- (b) The tangent at the point  $(1, a^3)$  passes through the point  $(\frac{1}{2}, 0)$ .  
Find the value of  $a$ , giving your answer in an exact form. [4]
- (c) By considering  $\frac{d^2y}{dx^2}$  show that the curve is convex for all values of  $x$ . [5]

**END OF QUESTION PAPER**

---

# OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.